MATHEMATICAL INVESTIGATION AND ITS ASSESSMENT: IMPLICATIONS FOR MATHEMATICS TEACHING AND LEARNING

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ABSTRACT

In view of the open-ended nature of a mathematical investigation (MI) and its emphasis on mathematical reasoning, problem solving, and communication, this paper proposes an analytic scoring framework and rubric for MI that assesses both its product and processes. The rubric underwent construct validation, try-out, and calibration by the raters and was found to be a fair and valid instrument for assessing MI. The study also proposes an assessment process for MI which includes the selection of raters, transmutation table, and procedures.

Keywords: *mathematical investigation, assessment in mathematics, analytic scoring rubric*

BACKGROUND

Suppose you were asked to investigate this situation: "Lines are drawn on a plane." Do you have a specific problem to pursue or a clear path to follow? Not likely. Such is the challenge of a mathematical investigation (MI) – a process-oriented mathematical activity that does not have a specific and recognizable goal or problem (Orton &Frobisher, 2005). It provides the students the opportunity to choose what aspects of the situation they would like to do and what strategies to use to search for patterns, pose a problem, and state, better yet prove their conjectures (Ronda, 2005).

The idea of MI encapsulates the calls for reform in mathematics education "to shift the learning of mathematics towards investigating, formulating, representing, reasoning and applying a variety of strategies to the solution of problems – then reflecting on these uses of mathematics – and away from being shown or told, memorizing and repeating" (NCTM, 1995).

The benefits of using MIs in the classroom are well-documented. MI develops students' mathematical thinking processes and good mental habits (Bastow, Hughes,Kissane&Mortlock, 1984; Jaworski, 1994; Orton & Frobisher, 2005), deepens the students' understanding of the content of mathematics, and challenges them to "produce" their own mathematics within their universe of knowledge (Ronda, 2005). Integrating MI in mathematics classes is also one way of encouraging schools to focus on the learner's reasoning, communicating, and problem solving skills and processes.

This paper presents a portion of a larger study which was conducted under a scholarship grant from the Department of Science and Technology-Science Education Institute (DOST-SEI) and a Professional Development Incentive Program (PDIP) grant from the Philippine Normal University-Manila.

But how does one assess the products and processes of MI, given its open-ended and complex nature? This question is at the center of this paper, a part of a larger study that described and analyzed the design, integration and assessment of MI in secondary mathematics (Nivera, 2008). The results of the study are particularly significant to mathematics education in the Philippines, where MI is in its infancy stage. Hardly any study in MI has been conducted locally.

Theoretical Framework

The theoretical orientation that underpins the research on the notion of investigations is the social constructivist theory that views mathematical learning as a social construction of knowledge from shared meanings (Ernest, 1991), where a student is not considered as an object but rather as a subject who continuously acquires more importance within his/her relationships with others. The quests and goals of a constructivist orientation are for students to take responsibility for their own learning, that is, to be autonomous learners, to develop integrated understanding of concepts, and to pose -- and seek to answer – important questions (Tobins&Tippins, 1993). By its very nature, this is the essence of doing MI.

This study views the development of mathematical processes such as reasoning, communication, operational, and recording to be at the heart of mathematical investigations. It defines MI as an extended project where students pose and work on their own problems from a given unstructured point of view, which focuses both on content and processes, and involves both a written and an oral presentation.

The reform agenda of mathematics education envisions a systemic change in all aspects of mathematics -- curriculum, instruction, and assessment. Figure 1 shows how the study envisioned all three components to promote the integration of MI in class.

For assessment to be congruent to curriculum and instruction, it has to engage students in meaningful tasks that address both mathematical content and processes and use performance tasks and scoring rubrics. Mertler (2001) and Moskal and Leydens (2000) provide useful ideas in designing a rubric that considers the issues on validity and reliability.

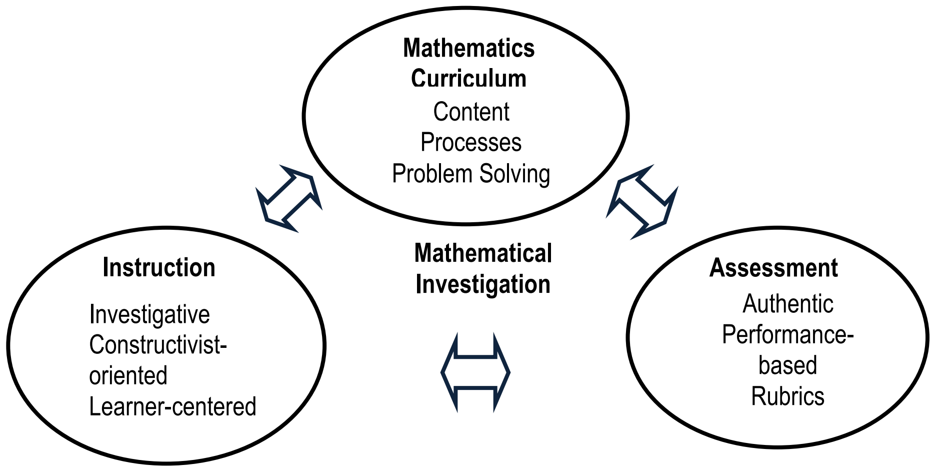

Figure 1. Congruence among Curriculum, Instruction and Assessment

This study adopted Peressini's and Webb's (1999) analytic-qualitative framework for analyzing students' responses to performance assessment tasks and modified it to reflect the facets of the constructs being assessed in MI using the ideas of Bastow *et al,* (1984), Jaworski (1994), Orton & Frobisher (2005) and Ronda (2005). The MI scoring framework became the basis for constructing the analytic scoring rubric for MI outputs. To calibrate the rubric, the sources of disagreements in the ratings were identified and the raters engaged in a negotiation process. Trice's (2000, in Mertler, 2001) ideas on constructing a transmutation table for a rubric became the basis for the transmutation table proposed in this study.

METHODS AND PROCEDURES

Method

Qualitative methods of data collection were used to provide a comprehensive documentation of the procedures undertaken to design and validate the analytic rubric and assessment process for MI. The results discussed in this paper were a part of a design-based study which tested in actual practice the conjecture on the interplay of factors in MI.

Design of the Study

The study was carried out in SY 2007-2008 in a public high school and involved two junior classes and their Geometry teachers. The students and their teachers were oriented about MI through separate half-day workshops. The researcher developed an analytic MI scoring framework and an analytic scoring MI rubric, which were evaluated by the teachers and students. The students experienced two cycles of MI, where each cycle began with group exploration of the task and ended with an oral presentation of their outputs. In both occasions, the teachers rated the product and processes of MI using the rubric. The ratings obtained by the different groups were analyzed for inter-rater reliability and used as basis for calibrating the rubric. The process of assessing MI products and processes was evaluated by the teachers and students in the focus group discussion (FGD).

Participants and Setting

The large public high school was located in a low middle-class area in Metro Manila. Two junior classes and their Geometry teachers were asked to participate in the study. Mark, another Geometry teacher, participated as an external rater. The students in each class were divided into groups of 5 or 6 and were a mix of high and low performing students. Two groups from each class were selected as case studies, their outputs subjected to careful analysis and assessment.

Scope

The study covered a period of 4 months, or about 2 grading periods. Data collection consisted of document analysis, focus group discussions, researcher's log, analysis of audio and video transcripts, and observations of students on task.

Tasks

The tasks used for the students' first and second MI are shown in Appendix A.

Procedure

The following steps were taken to ensure a valid assessment of the products and processes of MI: (a) identified the purposes of MI; (b) identified the facets of the constructs of reasoning, problem solving and communicating that might be displayed in MI, which would provide convincing evidence of the students' underlying processes; (c) prepared the scoring framework reflecting the facets of the constructs of MI to establish construct-related validity of the intended MI rubric; (d) developed the analytic rubric for MI based on the MI scoring framework; (e) identified the attributes that students needed to demonstrate in their MI output and clearly defined the characteristics or success indicators for each attribute or criterion to increase the rubric's reliability; (f) identified the three raters of the MI based on certain qualifications; (g) collaborated with the raters in validating the rubric in terms of its comprehensibility, usability and appropriateness for the purposes of MI; (h) tried-out the rubric on the first MI outputs; (i) calibrated the rubric through a negotiation process among the raters to identify and iron out discrepancies; (j) revised the rubric based on the raters' comments and suggestions; (k) tried-out the revised rubric on the second MI outputs; (l) conducted another negotiation process with the raters and obtained their comments and suggestions on the rubric, as well as the transmutation table for converting the rating obtained in the rubric to an MI grade; (m) computed the inter-rater reliability of the ratings given based on the rubric using intra-class correlation (ICC) coefficient; (n) obtained the students's comments and suggestions of the students on the rubric, the transmutation table, and the entire assessment process; and (o) revised and finalized the rubric and transmutation table based on the feedback of the raters and students.

RESULTS AND DISCUSSION

Purposely, MI output evaluates students' content knowledge and reasoning; hence both the product (i.e., the answer) and the process (i.e. the explanations) should be examined and assessed. This entails assessing both the written report and oral presentation. As such, issues on validity and reliability need to be addressed in the development of the rubric for MI.

Since establishing validity is dependent on the purpose of the assessment, it must guide the development of the scoring rubric (Moskal and Leydens, 2000). In this study, MI sought to promote and assess students' ability to analyze and reason mathematically, to use mathematical language, to communicate ideas, to apply mathematical knowledge to solve problems in a variety of contexts and discipline, and to assess content knowledge, including concepts, procedures, and skills. Clearly, an abundance of information would be lost by giving a single score to the MI using a holistic rubric. Thus, an analytic rubric that considered the unique characteristics of MI was prepared by the researcher.

To provide construct-related evidence for the rubric, the first step taken was to identify the facets of the constructs of reasoning, problem solving and communicating that might be displayed in MIs and would provide convincing evidence of the students' underlying processes. These included the students' ability to analyze the situation, search for patterns, state the problems, explain/justify conjectures, reorganize and extend ideas, and communicate these ideas orally and in writing. The second step was to carefully consider these facets in the development of the MI scoring framework and the establishment of scoring criteria.

The students' foundational knowledge, and their investigation and communication processes were identified as the main components of the MI framework, as shown in Appendix B. The component on foundational knowledge addressed the concern on content and included concepts, facts, and definitions, procedures and algorithms, and misconceptions, while the investigation process embraced the processes of analysis and reasoning, the communication process stressed the use of appropriate language, symbols/notations, and arguments.

The different parts and subparts of the analytic MI scoring framework were adopted as sections and subsections of the analytic rubric. Guided by the framework and Mertler's (2001) step-by-step process for designing scoring rubrics, the researcher identified the observable attributes that students needed to demonstrate in their MI output. The characteristics or success indicators for each attribute or criterion were clearly defined to reduce the occurrences of discrepancies between raters and to respond to the concern of intra-rater reliability. To ensure that consistency was maintained, the raters were instructed to revisit the established criteria throughout the scoring process. The final MI rubric, which was achieved after two try-outs, calibration, and validation by the raters and students, is shown as Appendix C.

To enhance the inter-rater reliability in an open-ended task like MI, at least 3 raters were employed. Thus, aside from the two subject teachers - Dang and Jason -- an external rater (Mark) was tapped. The three were asked to evaluate the rubric in terms of its comprehensibility, usability, and appropriateness for the task. At first glance, the raters found the rubric long and complicated because of the success indicators. Without the indicators, the rubric was found to be simple and user-friendly since the raters only had to tick 4, 3, 2, or 1 in each of the 15 criteria. However, the indicators for each score were necessary to guide raters on how a score of 4 differed from a score of 3, and so on. The indicators would also show the students what qualities were expected from their written outputs and oral defense. After a more thorough examination of the criteria and the corresponding success indicators, the teachers decided that these were comprehensible and self-explanatory, but that they could not really judge how usable and appropriate the rubric was for MI before actually using it in rating the students's outputs.

During their first MI, the students went through the MI stages of preliminary skirmishing, exploring systematically, making and testing conjectures, explaining or justifying, reorganizing, elaborating, and summarizing. They were asked to submit a written report of their MI with the following parts: introduction, statement of the problem(s) considered, mathematics used in the investigation (a short review, if necessary), presentation of the investigation (including conjectures, verification and proof), and possible extensions.

Both the raters and the students were given copies of the rubric ahead of time. As Moskal and Leydens (2000) explain, scoring rubrics must be shared with students in advance in order to allow them opportunity to construct the response, with the intention of providing convincing evidence that they have met the criteria. After all, students should not be held accountable for the unstated criteria.

Dang, Jason and Mark were given copies of the different groups' written reports two days before the oral presentation to give them ample time to scrutinize the outputs. After the oral presentations, they rated the students'

MI outputs using the scoring rubric. A summary sheet template was provided each rater for easier recording (This is shown as Appendix D).

The negotiation process among the raters showed that some degree of subjectivity came into play in rating the MI outputs even with the use of the rubric. There was disparity in the perspectives by which the raters judged the outputs and interpreted the criteria and success indicators. However, the findings showed that although the rubric did not completely eliminate variations between raters, it reduced the occurrences of these discrepancies. In fact, the slight differences in ratings did not affect the final rankings. The raters were still able to decide, with the help of the rubric, which group did the best work.

Essentially the same procedure was used in rating the second MI. Dang, Jason and Mark were again asked to rate the second MI outputs. They were joined by Helen, the head of the mathematics department, as one of the raters. Since her promotion to an administrative post, Helen had refrained from active classroom teaching. The results showed that Mark, Jason and Dang ranked the four groups in exactly the same way. Helen, on the other hand, ranked the groups differently. This finding brought to fore the need to choose the raters of MI carefully. The ratings given by Helen were excluded from the analysis.

The raters claimed in the FGD that the rubric was user-friendly and appropriate for assessing MI outputs since it contained the characteristics they were looking for in the students' MI. However, Mark raised his concern about the weightings of the rubric components. Since the focus of MI was the investigation process, he argued that it should get the most weight. The raters' comments and feedback served as basis for revising the rubric.

To check for inter-rater reliability, the intra-class correlation (ICC) of the ratings given by the three raters was obtained. Using two-way mixed effects model and a consistency type of computation, the ICC obtained on the first MI was 0.355 to indicate a moderate amount of variation among the scores given to each item by the raters. In the second MI, the ICC was 0.782, which indicated less variation among the scores. It could mean an improvement in the way the raters evaluated the outputs based on the descriptors in the rubric.

The transmutation table for converting the total MI score obtained from the rubric to an MI grade went through two revisions. The first one was found to be too favorable to the students. It was revised based on Mark's suggestion that since the highest possible score from the rubric was 60 and the lowest possible score 15, the lowest passing score should be raised from 27 to 38 to reflect 50% of the passing scores. Unfortunately, this revised transmutation table was observed to be too tough on the students much less seem to capture the level of their performance. Thus, it was revised twice, this time based on Trice's (2000, in Mertler, 2001) claim that in a rubric scoring system, there are typically more scores at the average and above average categories than below average categories. Thus, the final transmutation table has 30, almost two-thirds of the possible scores from the top, as the lowest passing score (see Appendix E).After studying the equivalent grades that the different groups obtained (using the final transmutation table), the teachers and students unanimously approved it.

In the FGDs conducted after the second MI, the students agreed that the rubric was a fair and valid instrument for assessing their MI outputs. With respect to the grades each group got in the first and second MI, everyone agreed that these were fair. As to whether each student in the group should get the same grade despite the differences in their contributions to the MI output, the students said 'yes'. They claimed that asking them to rate each other's contributions to the output would just lead to mistrust and infighting within the group.

Equally, the teachers found the entire MI assessment process fair, valid and reasonable.

CONCLUSIONS AND IMPLICATIONS

In sum, this study developed the following: (a) an analytic scoring framework for MI reflecting its purpose, constructs and processes; (b) an analytic scoring rubric for MI, whose construct-related evidence is established, and which has been tried-out and validated by the raters and the students; (c) a transmutation table for converting the MI score obtained from the rubric into a grade that teachers and students find fair and reasonable; and (d) a process of developing and calibrating a rubric through a negotiation process among the raters and the establishment of inter-rater reliability using intra-class correlation.

So, how does one assess MI outputs? First of all, qualified raters must be chosen to obtain a fair and unbiased assessment of the outputs. Each rater must have a good understanding of MI processes, products, and rubric; competent in both content and language; and must be disinterested or objective. Ideally, the students' MI outputs must be submitted to the panel of raters at least 2 days before the oral presentation to give them time to analyze and evaluate the write-ups. For the raters to judge the students' MI outputs rightly, they need to evaluate both the write-up and the oral presentation using the analytic rubric for MI. The oral presentation gives the raters insights into the rigor and depth of reasoning that went into the work, which students may have failed to reflect on their write-ups due to language difficulties.

For ease in rating several groups, each rater must be provided with a summary sheet template on which to write the scores for the different criteria in the rubric. The final scores are then converted into grades using the proposed transmutation table.

Finally, after the oral presentations, raters are encouraged to discuss with the students the strengths and weaknesses of their investigations for improvement in their succeeding work. The insights gained from MI must also be used by the teachers to guide classroom instruction.

The MI outputs in this study showed that students lacked skills in exploring and searching for patterns systematically, in writing problems and conjectures elegantly using correct and precise terms and phrases, in probing and proving the conjectures, and in explaining the salient features of their work to show their thinking and reasoning. By contrast, teachers needed further training in facilitating MI explorations, in proving, and in assessing the outputs.

Support from the administration was observed to be very crucial. Time pressures and fixation with the syllabus and the National Achievement Test and Division Test must be relaxed to provide the teachers and students more opportunities to do authentic tasks like mathematical investigations. The teaching of mathematics must include more activities that encourage pattern-finding, problem posing and problem solving, proving, reasoning, and communicating. More learner-centered modes of teaching and more open-ended assessment tasks must be encouraged.

Although this research provides some insights and mechanisms in assessing MI, many questions still remain unanswered and/or worth exploring:

- How does the integration of MI as an authentic assessment tool eventually influence classroom instruction as the teachers attempt to address the students' needs and weaknesses in conducting investigations?
- Are the MI rubric and assessment process presented in this study applicable to different school contexts and all kinds of MI tasks?
- How does the use of MI as an authentic assessment tool influence the teachers' and students' views towards classroom assessment and impact on the traditional national testing practices in the country?

Hopefully, the findings and outputs of this study have provided some initial, crucial steps towards encouraging the use of MI in the classroom and the conduct of further studies on MI in the Philippine setting.

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Appendix A
Mathematical Investigation Tasks

First MI Task

Sum of Consecutive Integers

Some numbers can be expressed as sum of consecutive positive integers.

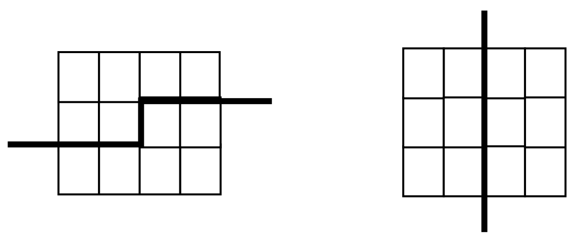
9 = 2 + 3 + 4
11 = 5 + 6
18 = 3 + 4 + 5 + 6

Investigate.

Second MI Tasks

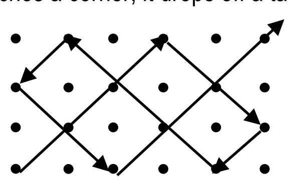
CUTS

Make a rectangular grid of squares. Cut it along the grid lines to make two identical pieces.Investigate.



BOUNCES

Imagine a rectangle on dot paper. Suppose it is a pool table.
Investigate the path of a ball which starts at one corner of the table, is pushed to an edge, bounces off that edge to another, and so on, as shown in the diagram. When the ball finally reaches a corner, it drops off a table.



Source: Bastow, B., Hughes, J., Kissane, B. &Mortlock, R.(1984). *40 Mathematical Investigations.* The Mathematical Association of Western Australia: Nedlands, WA.

Appendix B
Analytic Mathematical Investigation Scoring Framework

I. Foundational Knowledge – Foundation of mathematical working knowledge possessed by the students that is brought to bear on the mathematical investigation

A. Concepts, facts and definition
B. Procedures and algorithms
C. Misconceptions

II. Investigation Process – The analytical skills and reasoning abilities that the students demonstrate in doing the mathematical investigation.

A. Analysis – The mental operations and tendencies that students employ in the process of doing the mathematical investigation.

1. Understanding
    a. Attain familiarity with the situation.
    b. Look for possible patterns.
    c. State a problem.
    d. Examine special cases

2. Exploring
    a. Do a more organized and systematic search for patterns.
    b. Use tables, graphs or diagrams.
    c. Making Conjectures
    d. Testing/Verifying Conjectures

B. Reasoning – The modes of reasoning that the students exhibit while approaching and solving the tasks. (These include tendencies associated with the student's reasoning capabilities.)

1. Explaining/Justifying Conjectures
    a) Explain why the conjectures made will work for new or all cases
    b) Prove the conjectures (by mathematical induction, direct/indirect proof, visual proof)
2. Reorganizing
    a) Simplifying/generalizing the approach
    b) Justifying the connections among conjectures
        1. Extending

III. Communication – students' interpretation and understanding of the assessment task, and the corresponding expression of the analysis, reasoning and final conjectures (includes the summary of the report)

A. Language
B. Symbols/notation/diagrams/graphs
C. Argument (concise and logical)

Appendix C
Analytic Scoring Rubric for Mathematical Investigation (Final Version)

Criteria	Excellent(4)	Very Good(3)	Fair(2)	Poor(1)
I. Foundational Knowledge				
A. Concepts, facts and definitions				
1. Use of correct concepts, facts and definitions	Uses correct and appropriate concepts, facts and definitions	Makes 1 or 2 minor errors in the use of concepts, facts and definitions	Makes a major error or 3 or more minor errors in the use of concepts, facts and definitions	Has extensive errors in concepts, facts and definitions which make the entire investigation questionable or irrelevant.
B. Procedures and algorithms				
2. Selection and correct performance of appropriate procedures and algorithms	Selects appropriate procedures and performs all of them correctly	Selects appropriate procedures; makes 1 or 2 minor errors in computations	Makes one major error or 3 or 4 minor errors in doing the procedures or algorithms	Errors in carrying out the procedures or algorithms make the whole investigation questionable or irrelevant.
C. Misconceptions				
3. Absence of misconceptions	Has no misconception	Has (1) misconception	Has (2) misconceptions	Has (3) or more misconceptions
II. Investigation Process				
A. Analysis				
4. Range and depth of problem(s) investigated	Investigates at least 3 problems with commendable depth and rigor	Investigates at least 2 problems with satisfactory depth and rigor	Investigates at least 1 problem with satisfactory depth and rigor	Investigates one problem
5. Originality and complexity of problems investigated	At least 2 problems are not typical; shows originality and complexity	At least one problem is not typical; shows originality and complexity	Investigates only those problems that are simple and typical.	Investigates only problems that are exactly the same as those of others
6. Systematic study of problems	Explores the situation or problem systematically; Uses tables and diagrams	Explores the situation or problem in an organized manner	Explores the situation or problem with some ineffective system	Explores the situation or problem in a random and disorganized manner
7.Verification of solution or conjecture	Verifies the solution or conjecture by applying it to several cases; includes unusual cases	Verifies the solution or conjecture by applying it to several cases	Verifies the solution or conjecture by applying it to one case	Makes no or incorrect verification of the solution or conjecture
A. Reasoning				
8.Validity and depth of	Uses correct and valid reasoning and	Uses correct and valid reasoning	Has some minor flaws in reasoning	Has major flaws in reasoning

reasoning	shows depth in mathematical understanding			
9.Quality of the proof presented	Proves the conjecture convincingly using algebraic /analytic or correct and effective arguments	Proves the conjecture satisfactorily using correct arguments	Proves the conjecture using examples and diagrams or drawings	Fails to prove the conjecture
10.Ability to see connections	Makes significant connections with other problems or conjectures Extends the problems	Makes satisfactory connections with other problems or conjectures Extends the problems minimally	Makes minimal connections with other problems or conjectures	Makes no connections or extensions to the problem
III. Communication				
A. Language				
Criteria	**Excellent (4)**	**Very Good (3)**	**Fair (2)**	**Poor (1)**
11. Clarity of statements of problem(s) and conjecture(s)	States the problem(s) and conjecture(s) clearly, using precise and concise language	States the problem(s) and conjecture(s) clearly	States the problem(s) and conjecture(s) in a vague and incomplete manner	Does not state the problems nor the conjecture(s)
12. Clarity of written output of the investigation	Presents a complete, well-organized and clearly written output that includes a complete work trail	Presents an organized written output with an incomplete work trail	Presents a not so well-organized written output that shows an incomplete work trail	Presents a disorganized and incomplete written output
13. Clarity of oral report of the investigation	Reports the processes and results of the investigation clearly and comprehensively	Reports the processes and results of the investigation clearly – for the most part	Reports the process and results of the investigation in a disorganized manner	Does not report many of the processes and/or results
B. Symbols/notations				
Correctness of symbols, notations and labels	Uses correct and appropriate symbols, notations and labels	Makes minor errors in the use of symbols, notations and labels	Makes a major error in the use of symbols, notations and labels	Makes extensive errors in the use of symbols, notations and labels
C. Arguments				
Use of arguments in the written and oral report.	Provides sufficient, concise and valid arguments to support their reasoning and conclusions	Provides valid arguments to support their reasoning and conclusions	Uses some illogical and irrelevant arguments	Uses mostly illogical and irrelevant arguments or fails to provide any argument to support their reasoning and conclusions
For the teacher:	Other comments:			

Appendix D
Summary Sheet of MI Rubric

Criteria	Group 1	Group 2	Group 3	Group 4
1. Use of correct concepts, facts and definitions				
2. Selection and correct performance of appropriate procedures and algorithms				
3. Absence of misconceptions				
4. Range and depth of problem(s) investigated				
5. Originality and complexity of problems investigated				
6. Systematic study of problems				
7. Verification of solution or conjecture				
8. Validity and depth of reasoning				
9. Quality of the proof presented				
10. Ability to see connections				
11. Clarity of statements of problem(s) and conjecture(s)				
12. Clarity of written output of the investigation				
13. Clarity of oral report of the investigation				
14. Correctness of symbols, notations and labels				
15. Quality of arguments in written and oral report.				
Total Points				

Appendix E
Final Transmutation Table

Score	Equivalent Grade	Score	Equivalent Grade	Score	Equivalent Grade
60	100	45	87	30	75
59	99	44	86	29	74
58	98	43	85	28	74
57	97	42	85	27	73
56	96	41	84	26	72
55	95	40	83	25	72
54	95	39	82	24	71
53	94	38	81	23	70
52	93	37	80	22	70
51	92	36	80	21	69
50	91	35	79	20	68
49	90	34	78	19	68
48	90	33	77	18	67
47	89	32	76	17	66
46	88	31	75	16	66
				15	65