Number Theory Worktext for Teacher Education Program

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Abstract The lack of available reference materials for Number Theory course intended for the Filipino learners of the Teacher Education Institutions (TEIs) delimits the attainment of the program objectives in terms of content standards. This descriptive-developmental study evaluated and tested the effectiveness of the developed Number Theory Worktext for Mathematics students of Teacher Education programs. The purposively chosen Mathematics professors/instructors, graduate and undergraduate Mathematics major students in Bicol region evaluated the worktext in terms of content and coverage, theoretical considerations, appearance/visual appeal, and language used using the adopted assessment form for instructional materials. Results show that the worktext is valid and reliable as classroom instructional material and can significantly improve the performance of the students in the course. Further validation and periodic revisions of the materials shall be done to accommodate changes or updates in the Philippine Teacher Education program.

Keywords: development and evaluation, Number Theory, Mathematics worktext, Teacher Education Program

Introduction

Humans have a modest amount of innate ability to deal with numbers. Many distinct species can perceive the difference between two small numbers but this human innate capacity

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is certainly limited relative to the needs in our contemporary society. Mathematics as a cumulative science is much more than counting and simple arithmetic in which new results are built upon and depend on earlier results. Thus, throughout recorded human history, we find evidence of human effort to enhance their mathematical abilities by developing learning aids such as the counting board, abacus, Math tables, electronic calculators, and electronic digital computers (Moursund, n.d.).

Kim (1998) expressed his impressions that even those practicing mathematicians are frequently unaware of the principles behind Pythagoras' dictum that "All is number." This became one of the concerns in the study of Number Theory over the years. Number Theory as a course has been one of the acceptable required subjects for Mathematics major under the current curriculum of the Teacher Education program in the Philippines along with the neighboring ASEAN countries.

However, Chand (2015) identified that the unavailability of the proper facilities, poor standards of resources, negative attitudes of management towards the development of both human as well as material resources among others are some major global issues and concerns that hinder the delivery of quality teacher education programs. In a study on pedagogy, curriculum, teaching practices and teacher education; developing countries found out that there is weak evidence that the content and coverage of the teacher education curriculum is overloaded and irrelevant to rural or marginalized type of learners. Moreover, the same article reported that the Department for International Development found a limited number and of low quality textbooks as well as teaching and learning materials to support pedagogy in teacher education among the developing countries (Westbrook, et.al, 2013). Similar situations have been noted in the Philippine Teacher

Education institutions wherein several content courses have been offered with very limited reference instructional materials appropriate for the Filipino learners.

Tominez, Dela Cruz, and Gabatino (2013) recognized the importance of providing adequate, appropriate and varied instructional materials to concretize and substantiate learning. Teachers have the significant share in providing appropriate learning resources such as student materials, teacher support material, relevant technology, and an appropriate physical environment which provides the framework for the role of mathematics teachers in the curriculum development. The study of Edenfield (n.d.) in Georgia examined the selection, evaluation, and implementation of instructional materials by a group of teachers during their first teaching of their new integrated, process standards-based curriculum. The study indicated that teacher educators must help prospective teachers develop their knowledge and skills on curriculum development aligned to the set standards.

In fact, Mathematics majors of the Teacher Education Program in the Philippines experienced limited number of reference materials for Number Theory subject courses intended for the Filipino learners that are sold in any bookstore nationwide. In Bicol region, all its nine State Universities and Colleges (SUCs), together with other leading private Higher Education Institutions (HEIs), offered the Teacher Education program with Mathematics as one of the major fields of specialization.

The non-availability of references specifically for major courses in Mathematics of the Teacher Education program restricts the attainment of the objectives of the program in terms of content standards as stated in Section 6 of CMO 30 Series 2004 otherwise known as the "Revised Policies and Standards for Undergraduate Teacher Education Curriculum."These competencies include the basic and higher

level literacy, communication, numeracy, critical thinking and learning skills needed for higher learning. A meaningful and comprehensive knowledge of the subject matter for their teaching is required. Thus, professors in different fields of specialization of the Teacher Education programs should be innovative and creative enough in motivating learners to attain the required competency and standards of their career in the field of education.

The introduction of the K to 12 curriculum in the Philippine Basic Education may consequently result to changes in the Higher Education curriculum especially in Teacher Education program. Teacher Education Institutions (TEIs) offering Bachelor of Secondary Education (BSEd) then should provide appropriate teacher education for secondary education level in different fields. A wide range of appropriate content knowledge should be developed in preparation for their teaching in the future, which requires the availability of appropriate learning resources to serve as stimuli in the teaching-learning process.

Lipscomb, Swanson, and West (2004) expressed that appropriate, meaningful, and available instructional materials are among the facilitative tools, which support and scaffold student learning. Scaffold instruction characterizes the development and execution of instructional plans to lead the students from what they already know to a deep understanding of new material. Teacher adds supports and systematically build on students' experience and knowledge as they are learning new skills.

Culatta (2015) explicated the constructivist theory of Bruner that learners learn best if they are active participants of the learning process who are able to construct new concepts based on their current or past knowledge or schema. Hence, instructors should try and encourage students to discover principles by themselves through engaging activities.

They, as facilitators of the learning process should translate information and learning concepts into a format which suits the learners' present level of comprehension. The provision of an organized curriculum in a spiral manner is important so that students can continuously produce knowledge based on what they have already learned.

Moreover, the theory of schema emphasizes the importance of general knowledge for the formation of higher mental processes (Pappas, 2014). Schema, as a framework of preconceived ideas of the world, influences attention to and derivation of new knowledge since people can easily notice objects that fit into their schema, while verifying and re-interpreting those which contradictions the schema. Thus, teachers are tasked to help students develop new schemata and establish connections among them, which will improve their memory and understanding.

Both student teachers and graduate students of the Mathematics education already have preconceived ideas on the concepts, processes, and theories of numbers that need to be organized in a systematic way. These schemas may be further restructured in a deeper way so that they could have better insights and can be able to connect theories related to the study of numbers and number sense. Many new insights and innovations evolved in our modern times that would make the study of complicated concepts and processes easier by building on what students have already learned and mastered.

Appropriate selection of instructional media with due considerations of the students' schema can help facilitate an effective teaching based on the learning objectives. It can also provide positive impact in the improvement of students' learning, performance rate and achievement. There were several studies conducted on the importance of developing instructional materials that would aid teaching and can

contribute to the achievement of specific objectives of the subject suitable to the ability of students.

The quasi experimental study of Nwike and Catherine (2013) revealed that students taught with instructional materials performed better that those taught without instructional materials in the teaching of agricultural science. Ogaga, Wallace, and Benson (2016) also found out that the selection of relevant instructional materials; availability and ability of teacher to improve were significantly related to the teaching and learning of social studies in Secondary Schools in Oju local government area of Benue State.

Torrefranca (2017) subjected her developed instructional modules on Rational Expressions and Variations for second year high school students for evaluation by experts who strongly agreed that it satisfied the criteria that included objectives, content, format and language, presentation, and usefulness of the module. The module significantly improved the performance of the students in the posttest on the covered topics as compared to their pretest scores. Moreover, Cruz's (2015) evaluation on the developed worktext in Drafting Technology for Bachelor of Technology (BT) majors revealed that professors highly agree on the objectives, contents, activities, creativity, and evaluation procedures included in the text. The accuracy, completeness, and appropriateness of the worktext were rated from high to very high level, which indicates that it could be utilized and adopted for instruction.

The descriptive-developmental method of study of Terano (2015) evaluated the instructional materials in Differential Equations for Engineering program in terms of its contents and structure/format. The developed instructional material is a simplified text with workbook following the minimum requirement in accordance with the Commission on Higher Education (CHED) CMOs for Engineering

programs incorporating various aspects of learning: cognitive, psychomotor, and affective. It was found out that the material is highly acceptable for use as evaluated by the Engineering professors and students.

The aforementioned studies were found relevant and have some degree of similarity in the present study since they dealt on the development, evaluation and validation of the instructional materials for the intended learners. The present study dealt on the innovative instructional materials for Filipino Mathematics major students featuring the Number Theory course contents with the definitions, examples, drill exercises, and worksheets incorporating the minimum standards set by the Commission on Higher Education (CHED) for the BSEd degree program.

Purposes of the Research

The main purpose of this study is to develop and evaluate the effectiveness of Number Theory Worktext as an instructional material for Teacher Education programs. Specifically study sought answers to the following objectives: 1) develop the Number theory worktext; 2) establish the content and face validity and reliability of the developed worktext; and 3) determine the effectiveness of the worktext in improving the performance of the students.

Methodology

Research Design

This study used the descriptive-developmental research design in the systematic design, developing and evaluating the final product of instructional programs and processes. This research included several stages such as planning, development, dry-run, validation, and evaluation to achieve

the desired product. Also, one-group pretest-posttest design dictated the testing of the effectiveness of the Number Theory Worktext in improving the performance of the students.

Participants

There were three groups of respondents who participated to evaluate the developed Number Theory Worktext. The first group included 10 Mathematics professors/instructors of the Teacher Education program purposively chosen as the expert participants from the five State Universities and Colleges (SUCs) in Bicol region. These professors/instructors faced similar problems of looking for reference materials in the teaching of the course.

The second group of participants included graduate students taking Master of Arts in Education (MAEd) major in Mathematics of two SUCs in the region. These graduate students are professional Mathematics teachers employed in the different parts of Bicol Region as Mathematics teachers mostly in the High School level and some in the Tertiary Education level. Only those who received a copy of the materials were purposively chosen to evaluate the worktext.

Moreover, there were 17 BSEd Mathematics major students who participated as the subjects of the study in the one-group pretest-posttest experimental design. They were the only enrolled fourth year Mathematics major students of Teacher Education program of one State University of the region. They were also asked to evaluate the worktext.

Instrument

There were two main instruments used to gather the intended data in the study: (1) the Instructional Materials evaluation form (Appendix A); and (2) the Researcher-made test. The Instructional Materials evaluation form was used to determine the characteristics of the developed Number

Theory worktext adopted from SSC Memorandum Order No. 43, s. 2015 which was approved through the Sorsogon State College Board of Trustees (BOT) Resolution No. 27, s. 2016 otherwise known as "Policy Guidelines in the Evaluation, Production, and Publication of Instructional Materials."This instrument which has been utilized by the College since 2007 until its revisions in 2015 with Cronbach's alpha value of ($\propto$) of .83 has been found valid and reliable in evaluating instructional materials. It consists of the criteria for the evaluation of the characteristics of the instructional materials along content and coverage, theoretical considerations, appearance/visual appeal, and language used. This was used to determine the content and face validity of the developed instructional materials.

The researcher-made test is a 35 item multiple choice type of assessment test (Appendix C) which was developed based on the designed table of specifications (Appendix B) patterned to the course syllabus of Number Theory. Items included in the assessment test were from the test bank of the researcher since he started teaching the course. The test questions were all found valid and reliable with appropriate level of difficulty (ranges from .42 to .74) and index of discrimination (ranges from .22 to .45).

Procedures

There were two overlapping phases considered in this developmental method of study: (1) development phase, and (2) validation phase. The development phase follows the stages of planning, designing, and revising stage. The validation phase included the content and face validation by experts and students, reliability test through inter-rater agreement of experts, and the test for the effectiveness of the worktext in improving the performance of the students through the one-group pretest-posttest experimental design.

Development Phase

The planning stage was the preliminary activity of this phase in which the researcher designed a course syllabus for the course guided by the course description and outline as provided by CHED Memorandum Order (CMO) no. 30, s. 2004 and the revised Bloom's Taxonomy of the classification of learning objectives. The course syllabus defines the targets and expectations for the subject (Slattery and Carlson, 2005) which became the basis of the design in terms of contents and coverage, format and structure of the worktext based on the needs and context of the students. The materials were initially in the form of worksheets per content topic, which have to be accomplished by the students in the year 2014.

The first version of the Number Theory worktext was completed during the year 2014. Since content formation of the worktext was guided by the designed course syllabus for the subject, the topics were sequentially arranged. Each topic considered in the worktext contains discussions of the concept including the theorems and definitions with the attached worksheets/exercises. Discussions were made simple and brief as possible with due considerations to the students' needs and level of understanding. Grammar editing and proofreading of the worktext were eventually made to ensure that each topic is easily understood by the students and free from grammatical and/or typographical errors. This would serve as an available reference not only for instructors and professors teaching the subject Number Theory but also for BSED and MAEd Mathematics students. After the validation phase, the revised version of the worktext was made incorporating the comments and suggestions to improve the material.

Validation Phase

After the completion of the first version of the worktext, 10 Mathematics professors in the Teacher Education program of five SUCs in Bicol Region in the year 2015 have taken part in the evaluation of the developed Number Theory text as expert evaluators. These faculty members had the same problems faced by the researcher in looking for reference materials appropriate for the course. Each of them received one copy for utilization in their respective classes as part of the dry-run stage of the study. They were also asked to evaluate the worktext using the evaluation form during AY 2015-2016 ensuring that each of them has already utilized the material for instruction.

To triangulate the results of the content and face validation of the material, the researcher involved samples of MAEd and BSEd Mathematics major students in the region as potential users of the developed instructional materials. The MAEd students were from two SUCs in the Region who received and utilized a copyof the materials during the course study. There were 23 evaluation forms retrieved from this group of evaluators. Furthermore, the BSEd Mathematics major students were also asked to evaluate the worktext materials after a semester of its utilization as their reference material for the course. These BSEd students were subjected to experimentation using pretest-posttest design.

The researcher collaborated with the Mathematics professor in the Teacher Education program of one SUC in the region handling the Number Theory course for BSEd during the second semester of AY 2015-2016 to test the effectiveness of the worktext. The researcher conducted the one-group pretest-posttest experimental research design in testing the effectiveness of the worktext as part of its validation process. Seventeen enrolled Math major students took the pretest at

the start of the semester and the post test at the end of the semester to determine the scores gained.

Data Analysis

The study considered both the qualitative and quantitative analysis in the development and evaluation of the Number Theory worktext. During the design and development of the worktext, cross tabulation of the topics and competencies in the course has been considered to ensure that it captured all the required content standards and competencies. Appropriate statistical tools such as frequency count and weighted mean were utilized in the analysis and interpretation of the evaluation of the developed worktext by the three groups of respondents. These processes determined the level of content and face validity of the developed materials. Furthermore, F-test was utilized to determine whether there was a significant difference in the evaluation of the worktext by Mathematics professors/instructors, MAEd students, and BSEd students along its content and coverage, theoretical considerations appearance/visual appeal, and language used.

Furthermore, the inter-rater reliability was determined through the percent of agreement among the experts as demonstrated by Glen (2016) utilizing the Landis and Koch-Kappa's Scale for the strength of agreement. The strength of agreement per variable considered has been determined in the study. The t-test for dependent data was used to determine whether the posttest percentage scores of the students significantly improved as compared to their pretest scores after exposure to the worktext.

Results and Discussions

The subsequent sections provide discussions on the characteristics of the developed Number Theory worktext for the Mathematics major students under the Teacher Education

Programs in the Philippines. Table 1 presents the content topics of the developed Number Theory worktext for the Teacher Education program. The content topics included in the worktext are in accordance to the minimum requirements as directed by the Commission on Higher Education (CHED) in the Philippines.

Table 1. Content Topics of the Developed Number Theory Work Text for Teacher Education Programs

CHED Requirements	The Work text Material
I. Introduction to Number Theory	Topic 0: Introduction to Number Theory
	Topic I: The Number System
II. Divisibility	Topic 2: Divisibility
	Topic 3: Division Algorithm
III. Prime and Composite Numbers	Topic 4: Prime and Composite Numbers
	Topic 6: The GCD and LCM
	Topic 7: Prime Factorization
IV. Congruences	Topic 12: Modular Arithmetic [R]
	Topic 13: Coprime, Additive and Multiplicative Inverses [R]
	Topic 18: Linear Congruences
	Topic 19: Chinese Remainder Theorem [nR]
V. Euler's Function and Theorems	Topic 14: Arithmetic Function
	Topic 15: Euler's Theorems
VI. Fermat's Theorem	Topic 16: Fermat's Little Theorem
VII. Wilson's Theorem	Topic 17: Wilson's Theorem
VIII. Linear Diophantine	Topic 8: Euclidean Algorithm[R]
IX. Equation	Topic 9: Bezout's Theorem[R]
	Topic 10: Linear Diophantine Equation
X. Application	Applications
	Topic 5: Perfect Number [nR]
	Topic 11: Base Number System [nR]
	Topic 20: Quadratic Residue [nR]

Legend: *R* – required additional topic, *nR* – non-required additional topic

Source: CHED Memorandum Order No. 30, s. 2004

As gleaned from Table 1, there is an alignment of the CHED minimum required topics and the proposed topics in the worktext material. Enrichment has been made for every topic such as Introduction to Number Theory with the topic on the Number System. Division Algorithm has been included on Divisibility; the Greatest Common Divisor (GCD) and Least Common Multiple (LCM) together with the Prime Factorization were included as enrichments for Prime and Composite Numbers. With respect to Congruences, there were several enrichments made such as the inclusion of additional required topics on Modular Arithmetic together with Coprime, Additive, and Multiplicative Inverses aside from specific topic of Linear Congruences. Chinese Remainder Theorem has been included in the worktext as an additional non-required topic under Congruences. For Euler's Function and Theorems, Arithmetic Functions were intentionally introduced first as a general topic prior to Euler's Functions and Theorems. Euclidean Algorithm and Bezout's Theorem were the additional required topics prior to the discussions of Linear Diophantine Equations.

Additionally, both Fermat's Theorem and Wilson's Theorem were not given additional topics for they are already considered as specific topics which are the continuation of Euler's Theorem under the study of Number Theory. The worktext also features additional non-required (unrequired) topics as part of the enhancement of the subject course such as Perfect Numbers, Base Number System, and Quadratic Residue. These topics are non-required for the undergraduate Mathematics students but are required topics for the graduate students. The project component of the course, both written and non-written, included the applications of the topics which the students are required to submit before the end of the semester. The application of the topics varies from modulo art making, cryptography, and bar coding among others. These are included in the intended topics as part of the Outcomes-

Based Education (OBE) curricular program. Topics were arranged in spiral manner through which students are able to connect previous topics to the new topics applying additional algorithm skills.

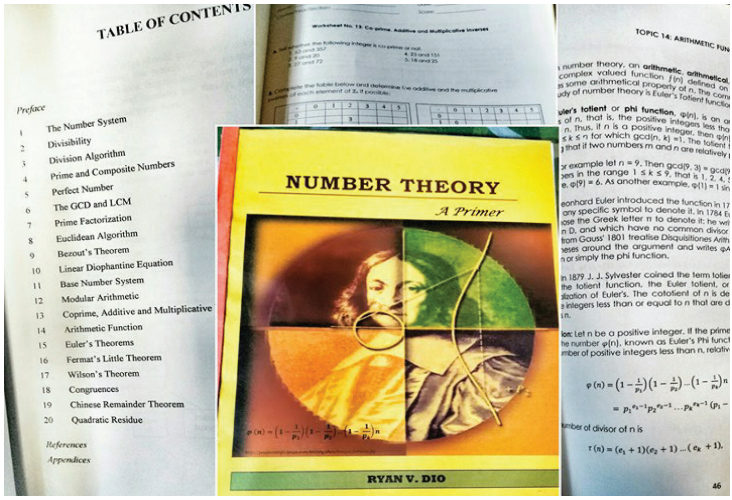

Figure 1. Screen Shots of Some Parts of the Number Theory Worktext.

The worktext contains the cover page (Figure 1), preface, table of contents, the content topics, references, and appendices. The content topic includes all the mentioned 20 primary topics of Number Theory. Inclusively, the developed wortext included discussions of the topic, examples, and worksheet exercises. Worksheet exercises vary from concept formation and recall, performing algorithm and solutions, problem solving, and proving. References include sources from printed materials and electronic sources. Information which supplements the teaching and learning of the content topics of Number Theory is included in the appendix section of the worktext, which also includes sample of student work on applications of Number theory.

This worktext maybe a usable reference material for the new curriculum of the Bachelor of Secondary Education (BSEd) major in Mathematics after the K to 12 transition period in the Philippines as reflected in the Commission of Higher Education Memorandum Order 75, series of 2017. The drafted BSEd curriculum by the Commission on Higher Education (CHED) includes the Number Theory course as one of the major subjects in which all of its required competencies can be found in the developed worktext material. This Number Theory worktext may serve as an available reference material for incoming first year college students after the SHS transition period who opted to enroll in the Teacher Education Program with specialization in Mathematics. Any Higher Education Institution (HEI) in the Philippines offering the BSEd program may adopt the worktext material intended for Filipino learners as it covers all the required competencies set by CHED in the subject course. This would answer the gaps and need for the available reference printed materials on Number Theory of the BSEd Mathematics major of the Teacher Education Institutions (TEIs) in the country.

Validity and Inter-rater Agreement of Experts

This section presents the results of the content and face validation of the worktext based on the assessment of the Mathematics professors/instructors, MAEd students, and BSEd students along content and coverage, theoretical considerations, appearance/visual appeal, and language. This portion also includes the comments and suggestions from the respondents that would support the quantitative results of the validation procedures under study. Results of the inter-rater agreement among experts are also discussed to ensure that the material has reached an intended validity and reliability level.

Table 2. The Perceived Validity Level of the Number Theory Work text

Characteristics	Weighted Means			F-Value	p-value
	BSEd (n=13)	MAEd (n=23)	Prof/Ins (n=10)		
Content and Coverage	4.77 O	4.57 O	4.63 O	1.586	0.218
Theoretical Considerations	4.85 O	4.65 O	4.80 O	1.486	0.239
Appearance/ Visual Appeal	4.85 O	4.51 O	4.65 O	3.259	0.049*
Language	4.90 O	4.41 VS	4.47 VS	5.359	0.009*
Over-all	**4.83 O**	**4.54 O**	**4.67 O**	**3.795**	**0.031***

Legend:*Significant at = 0.05, F crit = 3.245, O-Outstanding, VS-Very Satisfactory

Table 2 shows the summary of the perceived validity level of the Number Theory worktext along its content and coverage, theoretical considerations, appearance/visual appeal, and language used as assessed by the undergraduate students, graduate students, and professors/instructors. As reflected, the undergraduate students gave the highest over-all rating of 4.83 compared to the rating given by the graduate students (4.54) and professors (4.67), respectively, which are all interpreted as outstanding rating of the worktext. This result shows that the developed worktext captures the expected content coverage for the course with the use of appropriate language, which is very appealing to the Filipino learners as well as to the professors and instructors as users of the material.

The researcher solicited comments and suggestions from the three groups of respondents as part of the validation procedures of the developed Number Theory Worktext and to support the quantitative findings of this study. The Mathematics professors/instructors appreciated the worktext a lot for its clear content with sufficient examples and exercises that can be easily followed by the students.

Additionally, graduate students noted that the worktext is student-friendly and that the topics are well presented and sequentially arranged. These comments are also similar to the comments provided by one of the undergraduate students as follows:

> *"The book entitled 'Number Theory- a Primer' is a useful source of information for each required topic in the course Number Theory. The approach/style of presenting the lessons is well-organized and comprehensive."*

Table 2 also shows that the worktext is more appealing to the undergraduate students than the professors as revealed by the significant difference with $p<.05$ using F-test among the evaluation of the respondents on the characteristics of the worktext specifically on appearance/ visual appeal and language. This result further reveals that the appearance and language used in the worktext is more attractive to undergraduate Mathematics students than the other two groups of respondents. These findings could be supported by the feedback from the group of students who cited that they are visually attracted Number Theory worktext, which contains enough examples and exercises with the use of appropriate language intended for Filipino learners. Furthermore, the material is a reliable resource for learning that facilitates better understanding. One of their positive comments is as follows:

> *"There is sufficient amount of challenging activities which I can connect with and provide me motivation to strive and learn more in this major subject."*

Accordingly, the developed worktext would be able to provide better experiences for the Math major undergraduate students through the challenging questions and activities included in the text. This insight from the students indicates

that the worktext can be a good reference material which may contribute to better understanding of the concepts of the theories of numbers aligned to their needs and capabilities as Filipino learners.

Since the worktext is primarily intended for the BSEd students, the material is written in a language appropriate to the level of their understanding with due considerations of their prior knowledge. This makes the worktext user-friendly and more appealing to the Filipino learners as compared to the available online sources. The groups of graduate Mathematics students and professors/instructors have higher level of understanding on the component topics of the subject such that they perceived that the language can be improved in higher academic format or tone.

The respondents also gave their suggestions for the improvement of the text. One of the graduate students suggested to provide more examples from basic to complicated problems, examples which are supported with explanations and detailed solutions so that they can easily grasp the presented topics. It was suggested since some of the graduate students do not have background in some concepts being presented in the worktext. These concepts were not tackled during their undergraduate level that is why they found it difficult to study. Another suggestion given by one graduate student is to provide key to corrections for the exercises included in every topic of the text. Moreover, the undergraduate students suggested to include the Table of Binary Numbers and Hexadecimal Numbers with equivalent value in decimal numbers in the appendices of the text for easy reference. The students would like all questions included in the worksheet to have corresponding examples in the discussions. This suggestion is common to most of the students though one of the intentions of the worktext is to let them explore various learning strategies on their own given the basic information presented in the worktext.

Table 2 further suggests that the three groups of respondents have the same level of evaluation on the characteristics of the worktext along content and coverage, and theoretical considerations rated as "outstanding" in level. The worktext contains prerequisite topics of Number Theory course considering the prior knowledge of the learners that would lead to the formation of new generalized knowledge or theory in the study of the required competencies. Pappas (2014) expounded Bartlett's Schema Theory on students' generic knowledge including their background information and prior knowledge that aide the formation of mental representations. It is in this sense that teachers as well as every instructional material developed have a vital role to help students develop new generic schemata and establish connections among them.

The result of this study shows that the worktext has the quality to offer intended content and coverage of the course since there were enhancements provided with due considerations to the needs, interests, previous knowledge and level of comprehension of the Filipino students. This finding is similar to the result of the study of Cruz (2015) in which the professors of the Drafting Technology rated from high to very high level completeness and appropriateness of the developed materials, which is very essential for classroom utilization. This entails that the Number Theory worktext material has high level of content validity and face validity through the evaluation of the three groups of respondents.

In addition to these findings, Table 3 shows the results of the inter-rater reliability value of the Number Theory Worktext based on the evaluation of Mathematics professors/instructors. The worktext was evaluated along the content and coverage, theoretical considerations, appearance or visual appeal, and language used as part of the content and face validation of the material by the 10 Mathematics professors/instructors in Bicol region considered as experts

for teaching the Number Theory course. Results show that all of them provided a consistent rating of either agree (4) or strongly agree (5) None of these experts gave a "disagree" rating in all the indicators.

Table 3. Inter-rater Agreement among Experts

Worktext Characteristics	% of Agreement	
	Strongly Agree (5)	**Agree (4)**
1. Content and Coverage	58	5
1.1 Soundness of conceptual/ theoretical framework	(47)	(7)
1.2 Appropriateness of objectives	(62)	(2)
1.3 Correctness of information	(100)	(0)
1.4 Comprehensiveness of coverage	(47)	(7)
1.5 Sequencing of the lessons	(47)	(7)
1.6 Appropriateness and sufficiency of activities	(47)	(7)
Overall	**63% - Substantial/good**	
2. Theoretical Considerations	73	3
2.1. Suitability to students' level	(100)	(0)
2.2. Effectiveness as a learning mode	(80)	(0)
2.3. Attractiveness/appeal to learners	(33)	(13)
2.4. Educational significance	(80)	(0)
Overall	**76% - Substantial/good**	
3. Appearance/Visual appeal	62	2
3.1. Format/Layout	(62)	(2)
3.2. Packaging	(62)	(2)
3.3. Presentability	(62)	(2)
3.4. Over-all impact	(62)	(2)
Overall	**64%- Substantial/good**	
4. Language	22	42
4.1. Grammar and mechanics	(62)	(2)
4.2. Appropriateness of language	(2)	(62)
4.3. Clarity	(2)	(62)
Overall	**64%- Substantial/good**	

It is reflected in Table 3 that the overall percentage of agreement of the evaluator experts of 63% on content and coverage, 76% on theoretical considerations, 64% on

appearance and on language show a substantially good level of reliability. The data also reveal that the experts consistently *strongly agree* that the worktext material can provide correct information suitable to the level of intended learners. Most of the experts consistently provide *strongly agree* rating on the effectiveness of the materials which has educational significance among the Mathematics major students of the Teacher Education program since its objectives are appropriate and aligned to the intention of the course. Most of them also *strongly agree* on the appropriateness of the format/layout, packaging, presentability, grammar and mechanics, and the over-all impact of the appearance of the worktext. Moreover, most of the experts consistently *agreed* on the clarity of the material and the appropriateness of language used in the material.

The positive evaluation of the experts on the material is generally substantial/good as revealed by the high level of percentage of agreement consistent from agree to strongly agree. This result signifies that the worktext possesses appropriate level of reliability on its content and coverage, theoretical considerations, appearance/visual appeal, and language used based on the evaluation of the experts. This result confirms the reports of Albano (2016) who referred inter-rater reliability as consistency of the raters' evaluation on the qualities and/or characteristics of the material. This affirmed the fact that the developed worktext based on judgment of experts in the discipline, is a reliable source of information intended for the course.

Effectiveness of Number Theory Worktext in Improving the Performance of the Students

The effectiveness of the worktext was tested using the one-group pretest-posttest experimental design among the group of BSEd Mathematics major students in an intact class of one SUC in Bicol Region. This determined the

effectiveness of the developed Number Theory worktext in improving the performance of the students. At the end of the semester after the utilization and exposure to the developed worktext, posttest was conducted to determine whether there is a significant difference in the pretest percentage scores and the posttest percentage scores of the students. Table 4 shows the mean performance of the BSEd students during the pretest and posttest including the corresponding t-value for each topic.

Table 4 shows an over-all pretest result of 37.98% performance level which significantly increased to 77.82% in the posttest ($t = 16.999$, $p < 0.05$) after utilizing the developed and validated Number Theory worktext. This result reveals a high probability that the utilization and exposure to the developed worktext in Number Theory helped the Mathematics students learn the required concepts and led to better performance. This result resembles the findings of the study of Torrefranca (2017) where she used the developed instructional modules on Rational Expressions in teaching her students which showed a significant improvement in their performance in the posttest on the covered topics as compared to the pretest scores.

Table 4 also indicates the topics in which the students' performance significantly improved during the posttest as compared to the pretest. Students significantly improved on their performance in majority of the component topics such as the Number System, Divisibility, Division Algorithm, Bezout's Theorem, Linear Diophantine Equation, Modular Arithmetic, Coprime, Additive and Multiplicative Inverse, Congruences, Euler's Theorem, Wilson's Theorem, and Perfect Number. This result signifies that the scaffold instruction feature of the Number Theory Worktext helps students in improving their learning on the mentioned topics. The worktext includes worksheets that lead the students from what they already know to a deep understanding of

new material. Toh, Leong, Toh, Dindyal, Quek, Tay, and Ho (2014) also found out that the Practical Worksheet in Number Theory as an instructional scaffold helps pre-service mathematics teachers develop problem-solving dispositions alongside the learning of the subject matter.

Table 4. Effectiveness of Number Theory Work Text in Improving the Performance of the Students (n =17)

Topics	Mean Performance (%)		t-value
	Pretest	Posttest	
The Number System	37.65	80.00	10.182*
Divisibility	88.20	100.00	2.219*
Division Algorithm	0.00	64.71	5.416*
Prime and Composite Number	36.80	70.59	1.725
The GCD and LCM	47.06	70.59	1.461
Prime Factorization	64.71	70.59	0.324
Euclidean Algorithm	94.12	82.35	1.000
Bezout's Theorem	64.71	100.00	2.954*
Linear Diophantine Equation	29.41	82.35	6.628*
Modular Arithmetic	12.94	91.77	19.651*
Coprime, Additive and Multiplicative Inverse	11.76	61.76	6.733*
Congruences	47.06	90.20	5.416*
Arithmetic Functions	67.65	82.35	1.319
Euler's Theorem	41.18	94.12	4.243*
Fermat's Little Theorem	29.41	41.18	0.696
Wilson's Theorem	11.76	76.47	5.416*
Perfect Number	0.00	52.94	4.243*
Over-all	**37.98**	**77.82**	**16.999***

*Significant at = .05, t cri = 2.120

Generally there is a significant improvement in their performance in the posttest as compared to their pretest results. Though there was no significant difference detected, the students' performance during the posttest was still

very much higher compared to the pretest performance on Prime and Composite Number, the GCD and LCM, Prime Factorization, Arithmetic Functions, and Fermat's Little Theorem. In addition, these results revealed that this group of Mathematics major students already has prior knowledge about these component topics that resulted to non-significant higher gap of their performance during the posttest.

The study also revealed that emphasis given in teaching the content topics in the classroom situation leads to attainment of a more favorable result in the posttest. Along Euclidean Algorithm topic, the test item focuses on finding the GCD between the two numbers which can be computed using other strategies as already learned from elementary to high school level which made the students maintained their passing percentage of score from pretest to posttest. Results of this study imply that the students' prior knowledge and the utilization and exposure to instructional materials have a significant effect in the performance of the students. Thus, these should be considered in any innovation that the classroom teacher makes.

Conclusion

The main purpose of this study is to develop and evaluate the effectiveness of the Number Theory Worktext as an instructional material for Teacher Education programs. The developed Number Theory Worktext is in response to the specific minimum requirement along content standards set by the Commission on Higher Education (CHED) in the Philippines. The worktext has been rated with substantial percentage of agreement among experts from very satisfactory to outstanding level and thus was found as a valid and reliable material for the course.

The content and coverage, theoretical considerations, appearance/visual appeal, and language used in the material

are found outstanding by the undergraduate students, graduate students, and Mathematics professors/instructors in Bicol region. However, the worktext is more attractive to undergraduate Mathematics students than to the graduate students and professors in terms of its appearance and language. The three groups of respondents appreciated the worktext well for its clear and sequentially arranged content topics appropriate for the Filipino learners with due consideration to their previous knowledge learned. It was also revealed that the utilization of this user-friendly worktext for classroom instruction can significantly improve the performance of Mathematics major students in Number Theory required topics.

Findings of the study signify that the developed worktext would be able to provide better experiences for the Math major undergraduate students through the challenging questions and activities included in the text. This insight gathered from the group of students is one measure that the worktext may be classified a good reference material which may contribute to better understanding of the concepts of the theories of numbers aligned to their needs and capabilities as Filipino learners. This tool may answer the gaps and needs for available reference text materials in the Number Theory course as reflected in the new curriculum of the Bachelor of Secondary Education (BSEd) major in Mathematics major under Teacher Education Program after the K to 12 transition periods in the Philippines.

Recommendations

An interesting aspect of this study is that it is developmental in nature which is actually one of its weaknesses. The study utilized the three groups of respondents from the group of BSEd and MAEd students, as potential users, and the grouped of Mathematics professors as experts evaluators after the development of the material. These groups of

respondents may be considered as participants during the developmental phase so that every important feature of the materials will be considered from the very start. Only the group of Mathematics professors' as expert evaluators have been considered as region-wide participants during the dry-run stage and validation phase of the developed worktext. The experimentation using the one-group pretest-posttest design to measure its effectiveness was only done in one SUC which cannot be generalized the results for Filipino learners.

In order to generalize research findings, the developed Number Theory Worktext may be utilized for classroom instructions in the country and studies will have to be developed to test its effectiveness using various experimental research designs. The author should consider the comments and suggestions of the professors and students to further improve the contents of the worktext. Moreover, further validation and periodic revisions of the material shall be facilitated to accommodate changes or updates in the curriculum of the Teacher Education program. Submission of the material to the instructional review committee and/or interested book publishing companies may be facilitated for possible publications and wider dissemination to national level of Filipino learners.

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Appendix A

Instructional Material Evaluation/Assessment Form

(Revised Version of Evaluation Form based from SSC Memorandum No. 42, s 2006)

Date: _____

Author/Proponent/s:_____

Instructional Material: _____

Evaluator's Name: _____ *Position:* _____

__ __ __ __ __ __ __ __ __ __ __ __

Direction: Please rate the material per criterion on the 5–point scale: 1–poor; 2–fair; 3–satisfactory; 4–very satisfactory; 5–outstanding.

Criteria/ Indicator		5	4	3	2	1	Remarks/ Comments
1.	CONTENT AND COVERAGE						
	1.1 Soundness of conceptual/ theoretical framework						
	1.2 Appropriateness of objectives						
	1.3 Correctness of Information						
	1.4 Comprehensiveness of coverage						
	1.5 Sequencing of lessons						
	1.6 Appropriateness and sufficiency of activities						
2.	THEORETICAL CONSIDERATIONS						
	2.1. Suitability to students' level						
	2.2. Effectiveness as a learning mode						
	2.3. Attractiveness/Appeal to learners						
	2.4. Educational Significance						
3.	APPEARANCE/VISUAL APPEAL						
	3.1. Format/Layout						
	3.2. Packaging						
	3.3. Presentability						
	3.4. Over-all Impact						

4.	LANGUAGE						
	4.1. Grammar and Mechanics						
	4.2. Appropriateness of language						
	4.3. Clarity						

Other Comments: _____

_____

_____

_____

_____

_____

Signature of the Evaluator

Date:_____

Appendix B

Number Theory Test - Table of Specification

TOPICS	Total Number of Items	Placement of Test Items	Percentage (%)
The Number System	5	1,2,9,10,18	14.29
Divisibility	2	12,15	5.71
Division Algorithm	1	19	2.86
Prime and Composite Number	4	8,14,16,20	11.43
The GCD and LCM	1	6	2.86
Prime Factorization	1	3	2.86
Euclidean Algorithm	1	4	2.86
Bezout's Theorem	1	5	2.86
Linear Diophantine Equation	2	26,31	5.71
Modular arithmetic	5	7,13,23,25,34	14.29
Coprime, Additive and Multiplicative Inverse	2	22,27	5.71
Congruences	3	21,24,35	8.57
Arithmetic Functions	2	11,30	5.71
Euler's Theorem	2	28,29	5.71
Fermat's Little Theorem	1	17	2.86
Wilson's Theorem	1	33	2.86
Perfect Number	1	32	2.86
TOTAL	**35**	**1-35**	**100**

Appendix C

Researcher-Made Test in Number Theory

Name:_____ Score:_____

Course & Year: _____ Date: _____

Multiple Choice. Choose the correct answer by writing only the letter on the space provided before the number.

_____1. Which is NOT an integer?

a. -5 b. $^1/_5$ c. 0 d. $\sqrt{9}$

_____2. What property of equality is illustrated by $5 \cdot 426 \cdot {}^1/_5 = 426 \cdot 5 \cdot {}^1/_5$?

a. Reflexive property c. Commutative

b. Multiplication PE d. Associative

_____3. Which of the following is a perfect square number?

a. $5^{40}17^{24}$ b. $2^{31}3^{20}$ c. $5^{70}6^3$ d. 5^92^{30}

_____4. Given: a = 18 and b = 42. Find gcd (42, 18).

a. 2 b. 126 c. 6 d. 18

_____5. Find *u* and *v* such that $45u + 20v = \gcd(45, 20)$.

a. -1, 2 b. 1, -2 c. -1, -2 d. 1, 2

_____6. The product of two numbers is 2700. Their greatest common divisor is 15. What is their least common multiple?

a. 180 b. 120 c. 90 d. 45

_____7. Which of the following is equal to 17 (mod 5) ?

a. 20(mod5) c. 23(mod5)

b. 11(mod5) d. 32(mod5)

_____8. What is the sum of the squares of the first three prime numbers?

a. 14 b. 29 c. 38 d. 83

_____9. The set of whole numbers is equal to the following sets EXCEPT

a. $Z^+ \cup \{0\}$ b. $N \cup \{0\}$ c. $Z \cap N$ d. $Z - Z^-$

_____10. The number ¾ % is an example of rational number, write it as a decimal.

a. 0.75 b. 0.075 c. 0.0075 d. 7.5

_____11. What is the sum of the positive divisors of 32?

a. 31 b.32 c. 63 d. 64

_____12. Which of the following numbers leaves a remainder when divided by 9?

a. 369 b. 9,513 c. 10, 919 d. 84,843

_____13. Which of the following is equal to 8 (mod 5) ?

a. 2(mod5) c. 23(mod5)

b. 11(mod5) d. 37(mod5)

_____14. Which of the following is a mersenne number

a. 6 b. 7 c. 17 d. 129

_____15. What is the smallest number that can exactly divided by 2, 3, 4, 5 and 8?

a. 12 b. 20 c. 60 d. 120

_____16. Which of the following numbers is relatively prime with 18?

a. 24 b. 15 c. 25 d. 32

_____17. Let p be prime and $\in Z$. If gcd (a, p) = 1, then a^p?

a. a (mod p) c. a

b. 1 (mod p) d. 1

_____18.Which of the following is a multiplicative inverse of $^{1}/_{9}$?

a. – 9 b. 9 c. $-^{1}/_{9}$ d. 81

_____19. What is the remainder when -278 is divided by 12?

a. -24 b. 10 c. -2 d. 9

_____20. How many integers between 20 and 30 are prime?

a. Two b. Three c. Four d. Five

_____21. If $2x \equiv 3 \bmod 5$, which of the following can be the value of x?

a. 0 b. 2 c. 4 d. 5

_____22. What is the additive inverse of the element 5 in Z_7?

a. -5 b. -2 c. 0 d. 2

_____23. Find -371 (mod 8).

a. 3 b. 4 c. 5 d. 6

_____24. If $b \equiv a \pmod{M}$, which of the following is its equivalent value of a(mod M)?

a. b + a b. b – M c. a + M d. aM

_____25. Which of the following is the multiplicative inverse of 4 in Z_7?

a. 0 b. 1 c. 2 d. ¼

_____26. Which of the following Diophantine linear equation has integral solutions?

a. $2x + 4y = 1$ c. $10x + 5y = 12$

b. $2x + 5y = 1$ d. $9x + 6y = 2$

_____27. How many elements of Z_{12} has its multiplicative inverse?

a. 3 b. 4 c. 8 d. 12

_____28. $2^7 (\text{mod } 5)$ = _____.

a. 3 b. 5 c. 128 d. 640

_____29. Find the remainder when 2^{99} is divided by 7?

a. 1 b. 2 c. 4 d. 5

_____30. How many positive divisors 180 have?

a. 8 b. 15 c. 18 d. 24

_____31. Find the general integral solutions to $2x + 3y = 7$.

a. (7 + 2k, -7 + 3k) c. (7+3k, -7 +2k)

b. (-7+3k, 7 -2k) d. (7+ k, -7+k)

_____32. Which of the following is an example of perfect number?

a. 2 b. 4 c. 18 d. 28

_____33. What is the remainder when 10! is divided by 11?

a. 2 b. 5 c. 8 d. 10

_____34. Evaluate: 9 – 15 in modulo 8?

a. -48 b. -6 c. 2 d. 6

_____35. Which of the following linear congruence has solutions?

a. $5x \equiv 7 \text{ mod } 15$ c. $5x \equiv 10 \text{ mod } 20$

b. $2x \equiv 5 \text{ mod } 8$ d. $4x \equiv 2 \text{mod} 12$

——————— *G O O D L U C K !* ———————