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Learning Styles and Cognitive Processes of Students in Performing Problem-Solving Tasks in Mathematics

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ABSTRACT The study aimed to determine and describe the cognitive processes and learning styles of 12 eight graders in performing problem-solving tasks. The cognitive processes of students with different learning styles were studied in terms of the cognitive behaviors they manifested as they were given problem-solving tasks. These cognitive processes were understanding the tasks, specializing, generalizing, conjecturing, justifying, and looking back. Forty-one of the original 75 students from two sections of a government-owned secondary school in Quezon City were classified as active, reflective, sequential, and global, based on learning styles as revealed by the Felder and Soloman's Index of Learning Styles Questionnaire. Consequently, three students from each learning style were randomly chosen as participants of the study. The three phases of the study involved audio taping the students while doing the problem-solving activities and interviewing them afterwards, coding all the transcribed scripts while analyzing students' problem-solving processes, and methodological triangulation in order to come up with a conclusion of the cognitive processes done by students of different learning styles. The findings revealed that cognitive processes in problem

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solving were manifested variedly by the four types of learners in terms of the observed cognitive behaviours. Finally, some pedagogical implications were cited in order to make the teaching of problem solving more functional.

Keywords: Learning styles, cognitive processes, problem solving task

Introduction

Problem solving is defined as an activity that requires one to find a solution to a given problem situation. Mathematics educators believe that problem solving is not just a vehicle for following some rules in order to obtain a correct answer. Instead, problem solving is believed to be a process wherein one decides from logical deduction what algorithms to apply and evaluates the results obtained from applying the chosen algorithm. Thus, it is imperative to check if the answer obtained is reasonable, if the process done is the most efficient, and if the ideas discovered and algorithms applied may be extended to a more complicated problem situation thereby laying a venue to create new knowledge. These skills are important in order to survive in a challenging society where one is always confronted with problem situations of varying degrees of difficulty. In other words, skills in problem solving should be developed because of its inevitable role in the society. Thus, mathematics teachers are challenged to expose students to problem solving tasks so that mathematics teaching would be more functional in the society. In fact, the National Council of Teachers of Mathematics (NCTM, 1980) recommended that problem solving be the focus of mathematics teaching because, they say, it encompasses skills and functions which are an important part of everyday life. However, different types of students view and solve mathematics problems differently as humans decipher and solve real-life problems, whether in their careers or in other aspects of life, differently as well.

Felder and Silverman (1991), in their dimensions of learning styles, distinguish active, reflective, sequential, and global learners. *Active learners* tend to retain and understand information by doing something active in it; they tend to be experimentalists; and they are the ones who evaluate the ideas, design, carry out the experiments, and find the solutions that work—the organizers, the decision- makers. On the other hand, *reflective learners* do not learn much in situations that provide no opportunity to think about the information being presented; they tend to be theoreticians; they are the mathematical modelers or the ones who can define the problems and propose a possible solution. Furthermore, *sequential learners* follow linear reasoning processes when solving problems; they can work with a material when they understand it partially or superficially; they may be strong in convergent thinking and analysis; and they learn best when a material is presented in a steady progression of complexity and difficulty. Conversely, *global learners* make intuitive leaps and may be unable to explain how they came up with solutions; they may have great difficulty in working with a material when they understand it partially or superficially; they may be better at divergent thinking and synthesis; they sometimes do better by jumping directly to more complex and difficult material; and they are the synthesizers, the multidisciplinary researchers, the systems thinkers, the ones who see the connections no one else sees.

The foregoing revealed that students of different learning styles take in and process information and deal with problem-solving tasks differently. Zhang and Sternberg (2006) defined learning style as part of one's intellectual style and as a person's preferred way of processing information and dealing with tasks. With these, they concluded that, at some degree, learning style is cognitive, affective, physiological, psychological, and sociological. Thus, it is worthwhile to link learning style with cognitive processes that learners manifest in dealing with problem-solving tasks.

In the Philippine setting, the Department of Education (2013) stressed that the twin goals of mathematics in basic education are critical thinking and problem solving. These goals are achieved by developing students' specific skills and processes such as knowing and understanding; estimating, computing and solving; visualizing and modeling; representing and communicating; conjecturing, reasoning, proving and decision making; and applying and connecting. Additionally, the Department of Education (DepEd, 2013) encouraged teachers to use cooperative learning in developing such skills and processes. Accordingly, cooperative learning is an active strategy that is achieved by grouping students so they could work with their fellow students as they all engaged in a shared task. There are different ways of grouping students, and one useful way is to group them based on the aforementioned types of learners. Analyzing the students' learning styles and grouping them based on the result of the analysis may aid the teacher in determining the specific needs of each group. Consequently, mathematics teachers may match their learning resources and teaching strategies based on the learning preferences and needs of students. If students would be taught in a manner they prefer and based on their needs, then these processes might lead to students' increased comfort level and willingness to learn which may result in being able to solve more problems successfully. Apparently, students' skills in problem solving may be developed to the fullest and ultimately, teaching mathematics may be more effective and functional.

In light of the above, the researchers conducted a case study to identify the cognitive processes manifested by

Grade 8 students of different learning styles. It is hoped that the study would motivate mathematics teachers of the same grade level to initially adjust their teaching practices based on the cognitive processes and learning styles of students that would be identified in this study, and eventually identify cognitive processes as may be manifested by their own students with the corresponding learning styles and match them with their teaching styles and the learning resources that may be beneficial to their students. Finally, it is hoped that mathematics teachers in other grade levels would be motivated to conduct a parallel study for more positive results that may help improve mathematics education in the country.

Purpose of the Research

This research aimed to determine and describe the cognitive processes and learning styles of Grade 8 students in performing problem-solving tasks. Specifically, it sought answer to the question, how are the cognitive processes manifested in terms of cognitive behaviours while students of different learning styles are performing problem-solving tasks?

Framework of the Study

On Learning Styles

Felder and Silverman's dimensions of learning styles made a distinction between active, reflective, sequential, and global learners. *Active learners* prefer to try out something first to see how it works. They tend to retain and understand information best by doing something active with it—discussing or applying it or explaining it to others. In contrast, *reflective learners* prefer to think things quietly first.

They prefer working alone unlike active learners who tend to like group work. As for sequential learners, they tend to gain understanding in linear steps, with each step following logically from the previous one. On the other hand, global learners tend to learn in large jumps, absorbing a material almost randomly without seeing connections, and then suddenly "getting it."

On Cognitive Processes and Problem Solving

Psychologists study how people process information using problem solving tasks (Ciccarelli & White, 2010). The cognitive processes involved in processing information may be described in terms of the observable traits the student manifests while performing problem solving tasks. Mason's, Burton's, and Stacey's (2010) theory of thinking mathematically emphasizes three phases in problem solving, namely entry, attack, and review.

The *Entry phase* can be described as the "understanding stage" in which the solver reads the question carefully and decides what exactly he or she wants to do. Furthermore, this phase may be described in two ways: by absorbing the information given and by finding out what the question is really asking.

When the solver fully understands the question, he or she enters the *Attack phase*. In this phase, it is normal to be stuck, and the solver must accept it because it is one way of learning how to get unstuck. The mathematical processes involved in this phase are specializing, conjecturing, and justifying.

As defined by Mason's (2010) group, *specializing* is the process of randomly selecting examples and trying them. This process is also described as "getting the feel of the question", and it serves as a preparation for generalization. "*Generalizing* starts when one senses an underlying pattern, even if he cannot articulate it." The generalization can lead to conjectures (what), justification of these conjectures (why), and finding where the conjectures are likely to be true.

Stacey (2005) described *conjecturing* as the "prediction stage" where the solver predicts the relationships of the given data and search patterns to come up with reasonable results. After formulating some conjectures, the solver tests them and decides to retain or refute the conjecture—this is the *justifying* process. The *review phase* can be associated with Polya's (1957) looking back phase, where the solver checks what is done and reflects on the key concepts to extend the process of solving.

Based on the theories discussed above, this study adheres to the belief that students process information differently since they have different ways of learning. Cognitive processes in solving problems may vary. Each type of learner may manifest all the cognitive processes, but the processes will differ in terms of the cognitive behaviours exhibited by the students as they perform the problem solving tasks. These points are summarized in the paradigm drawn below.

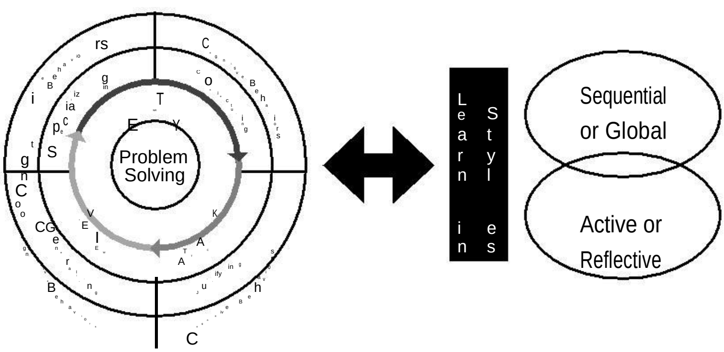

Figure 1. Framework of the Study

Methodology

Research Design

This research used the descriptive case study design. It described the learning styles and cognitive processes of selected students in performing problem-solving tasks. Further, it focused on the students' insights and the researchers' observations and descriptions of data based on the models presented by some educational psychologists.

Participants/Respondents

The 12 participants of this study were chosen from two sections of a government-owned high school in Quezon City. Initially, there were 75 students whose learning styles were assessed using Felder and Solomon's index of learning style. Results yielded that only 41 students are active, reflective, sequential, and global learners. Out of 41, three students from each type of learner were chosen at random to participate in the study.

Instruments

The researchers made use of the 44-item Felder and Solomon's index of learning style questionnaire to assess the participants' learning styles. This instrument, available online, requires a person to complete an open statement on a learning scenario. Also, a set of guidelines is provided if the answers in the questionnaire would be manually assessed.

Further, a validated five-item problem solving test (Appendix) was utilized to elicit the students' cognitive processes. The said test consisted problems on age, mixture, money, and work that may be solved using the concept of linear equations in one variable. Finally, observation notes and interview guides were also designed to triangulate the data gathered by the scorers.

Data Collection and Analysis

The learning style of each respondent was pre-determined following the set of guidelines provided by Felder and Solomon. Each respondent was presented with the problems one at a time and was asked to solve each problem using the think-aloud protocol with no interruption (unless needed) . This scheme allowed the students to say what they were thinking, seeing, and feeling while solving the problems. The problem-solving activities were audio-taped and transcribed. Consequently, two math teachers were invited to code the transcriptions using the coding scheme of Yeo and Yeap (2009) in order to elicit the students' cognitive processes. They were initially trained to do the coding prior to the transcription activity. Students were then interviewed to talk about the problem-solving activities and triangulate the codes of the teachers. Data were then summarized to identify the cognitive processes/behaviours manifested by active, reflective, sequential and global learners.

Results and Discussion

Table 1 summarizes the cognitive processes, cognitive behaviours and indicators in each phase of the problem-solving tasks, and the types of learners who manifested the corresponding behaviour. The discussions that follow are specific descriptions on how the learners manifested such behaviour.

Table 1. Cognitive processes manifested by four types of learners

Phase	Cognitive Process	Cognitive Behaviour	Indicators	Type of Learner
Entry	Understanding the task	Reading the task	Quietly read the problem	Active, Reflective, Sequential, Global
		Highlighting key information	Wrote the given on the paper	Active, Reflective
		Rephrasing the task	Translated words or phrases in Filipino or paraphrased containing words from the original task but must be preceded by phrases such as "in other words" or I must do this or that"	Active
Attack	Specializing	Thinking of a plan	Made connections on what they know and what the problem was asking to do or laid possible plans to solve the problem	Active, Reflective, Sequential, Global
		Decided on a plan	Implemented the plan	Active, Reflective, Sequential, Global
		Visualizing information	Drew a diagram or used tables and other representation to illustrate the problem	Active, Reflective, Sequential
		Rereading the task	Reread the task or parts of the task to think of a plan or monitor the progress	Active, Reflective, Sequential, Global
		Using Algebra to execute the plan.	Used variables and other symbols to solve the problem	Active, Reflective, Sequential, Global
		Performing calculations to execute the plan	Used fundamental operations in solving	Active, Reflective, Sequential, Global
		Using reasoning	Reasoned out or explained the solution/ steps done in solving	Active, Reflective, Sequential, Global
	Conjecturing	Observing and implementing pattern	Manipulated the given information to come up with reasonable answer	Active, Sequential, Global
	Justifying	Verifying the pattern	Performed calculations to verify the pattern	Active, Sequential
Review	LookingbackGeneralizing	Summarizing the solution	Wrote a conclusion; Used a statement to answer the question/ do the task in the problem	Reflective
		Checking, reflecting, or extending	Checked if the solution is correct Reflected on the reasonableness of the answer. Extended the problem to other situations	Reflective

On Understanding the Task

Table 1 shows that the problem-solving activity of all types of learners started in the *Entry phase* where the students spent time understanding the tasks. Although the learners were instructed to apply the think- aloud protocol, all learners chose to *read the problem silently*. For most of them, reading the problem on their own allowed them to make sense of the tasks they needed to do. As active and reflective learners, they *highlighted key information* in the problems, but the techniques in doing so varied from learner to learner. For instance, the three active learners pointed their pens to “P3.75” on the problem and wrote that amount on their questionnaires as they reiterated “ *Di ba 3.75 yung pen, ‘tas five times raw yung mabibili nya.”* (Each pen costs 3.75, then he can buy five times [Code: pointed ball pen to ‘5 pens’ on the questionnaire]. Consequently, active learners were able to formulate ideas on how to progress in understanding the problem. During the problem-solving activity, they realized that rephrasing could help them further understand the problem. This task is consistent with what was pointed out by Felder and Silverman that active learners prefer to try out something first to see how it works. They tend to retain and understand information best by doing something active with it; that is, applying it.

On the other hand, instead of pointing to some relevant information given in the problem as what was done by the active learners, all three reflective learners wrote the initials of the names of the persons and the cue words/concepts in the problem such as “work”, “rate”, and “time”. This act suggests that while they were reading the problem, they were already formulating initial plans, including the steps they had to do and the methods of solving they had to apply in order to obtain the answers. This is consistent with one characteristic of reflective learners as mentioned by Felder and Silverman, that is, reflective learners prefer to think things quietly first. However, two of the three reflective learners focused on the drawing tables instead of just writing the given on the paper while they were highlighting the information. This indeed was immediately followed by drawing some figures that helped them to concretize the problem situation. Tabulating the given and unknown information suggests that reflective learners have already decided what plan they would apply to solve the problem.

On the contrary, global learners did not highlight key information. It seemed that while reading the problem, they were already retrieving some important information in order to come up with a plan. In fact, during the interview, a global learner said, "*Pag po binabasa ko 'yong problem, nire-recall ko po 'yong mga na-solve namin na parang pareho din po ang gagawin, at tsaka po iniiisip ko na rin po ang formula.* " (While reading the problem, I am recalling those problems that have the same solution, then I am also thinking of the formula.). This affirms one characteristic of a global learner as mentioned by Felder and Silverman, that is, global learners tend to learn in large jumps and then suddenly "getting it." These observed cognitive behaviors in the process of understanding the task led all types of learners to decide to either try to solve the problem or to abandon it.

On Specializing, Conjecturing, and Justifying

Specializing, conjecturing, and justifying were the cognitive processes manifested by all types of learners in the *Attack phase* of the problem-solving process. In *specializing*, active learners laid their plans and solutions to answer the problem. It may be noted that two active learners repeatedly said, *"Alam ko po meron itong P, r, b. Bale, ganun, ganun. Yung triangle, Ma'am. Yun ba yun, Ma'am?"* (I know that it involves P, r, b, those that we can put in a triangle, right, Ma'am?) [Code: Asked questions to verify if what she knew was correct.] This cue indicates that active learners were already thinking of a plan to solve the problem by making connections between what they know and what the problem was asking. Furthermore, the three active learners thought, *"Divide natin. So ano, kapag dinivide yung 25 thousand... So 12 500."* (Let's divide. So, if we divide 25 thousand... It yields 12 500.) [Code: Plan: partition the amount into two; multiply the two amounts with their corresponding rates and time; the sum should be P900.00.] This response suggests that the active learners decided on a plan to solve the problem, and their plan included solving by reasoning and performing some calculations. Consequently, they executed their plan that enabled them to get the correct answer. Moreover, it was also very evident that the active learners were rereading the task to think of a plan to solve a problem. This cognitive behaviour was seen as the three active learners unanimously pointed out, "*"Nag-invest raw po si Mr. Santos ng certain portion."* (Mr. Santos invested a certain portion for two years that offered a simple annual interest. [Code: Rereading to plan a solution and monitor her understanding of the problem.] In implementing the decided plan, one active learner used Algebra and performed calculations/applied the four fundamental operations in solving while the two others resorted to performing calculations and using reasoning when they could not solve the problem algebraically. Visualizing information was also observed in the solutions of the active learners. They used triangular models which helped them visualize the problem situation and analyze the relationship of the variables as they drew a triangular model for problems that involved percentage, rate, and time and those that involved work and investment.

Similarly, reflective learners used mnemonic model to recall the formulas and important algebraic processes needed to solve the problem. Thus, it may be inferred that the reflective learners were able to determine if they can solve the problem or not. For instance, during the interview, one reflective learner said, "*Ano po… hindi ko na lang po si-nolve 'yong problem 2 kahit alam ko po kung ano ang gagawin, kasi po hindi po nag-fit sa triangle. Nakalimutan ko po ang formula e.*" (I did not anymore solve problem number 2 although I knew what to do because it does not fit the triangle. I forgot the formula.). This response suggests that the reflective learners' fixation with the formulas, deemed unsuccessfully recalled, led them to decide not to answer the problem even if there were other means to do so. They easily predicted that they could not solve the problem anymore instead of thinking of other ways to solve it. These observations confirm the theory of Mason's group that when a solver enters the *Attack Phase* , it is normal to be stuck. Notably, the same act was also observed with the global learners. After retrieving important information to come up with a plan, the global learners decided whether to abandon the tasks (because they simply did not know how to solve it) or to think of another means of solving if working with algebra was difficult. For example, two global learners did not solve the problems alone and mentioned, "*Hindi ko po masolve. Dati po kasi tumutulong sa akin si teacher pag hindi ko maalala ang formula.*" (I could not solve it. I am used to be guided by our teacher if I could not recall the formula.). This could be a result of fixation. Thus, it may be inferred that reflective and global learners decided not to answer some of the problems because they could not recall the formulas and other related algebraic procedures on their own.

Sequential learners, however, immediately drew some figures after tabulating the given and unknown quantities. Shown below is a sample of a student's work with a tabulation of the given and a figure drawn that shows the relationship of the variables used.

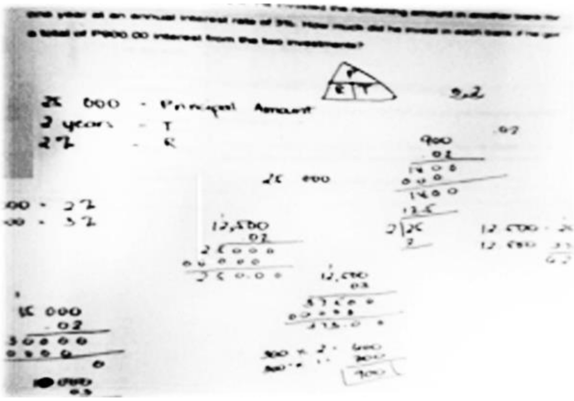

Figure 2. Sample work of sequential learner 1

Figure 2 shows that sequential learner 1 drew a figure after tabulating the known and unknown quantities. This style may indicate that the learner would want to see the relationship between the variables that he/she used. The illustrations served as a guide in working step by step towards the correct answer. Likewise, sequential learners used algebra, rephrasing the tasks and performing calculations to back up their plan and monitor their understanding and progress. These are cognitive behaviors on *specializing*. For instance, two sequential learners said, "*Yong amount of water po dito sa table ang x, at ito po (pointing to the figure) ang water. Then ilan daw po ang ia-add* [water] *nyo based on acid po, kaya yung water 0%* [on the second box] *equals* x + *500.*" (From the table, x is the amount of water. Then, you are asked how much water will be added based on acid, so the water is 0% equals x + 500.).

Conjecturing and justifying were manifested by two active learners. Interviews with them revealed that they searched for patterns while reading the problem. They stated, "*Ano po…. may pattern po akong nakita… divide 'yong sum of the ages by 3 para i-represent 'yong age ng* 1." (I saw a pattern... divide the sum of the ages by 3 to represent the age of person 1.) Implementing the pattern they discovered actually led them to the correct answer in just one problem. From their solutions, it was observed that they discovered that the pattern they formulated could not be applied to all similar problems. When the pattern worked for the case they were dealing with, they found it hard to believe that the solution was wrong and that the pattern they used that led them to the correct answer in one problem is not applicable to all cases. This was evidently observed particularly when they reiterated, "*Kaya lang po, hindi na nag work 'yong pattern sa ibang problem. Bakit po ganon?*" (But the pattern did not work in some problems anymore. Why is that so?) This act indicates that they found it difficult to accept that their conjecture could not be applied to all cases after exerting a lot of effort discovering it, and they became less critical to check if it actually works in the problem they were dealing with.

Similar to active learners, using algebra and performing calculations were manifested by reflective and global learners to execute the decided plans to solve the problems. This is evidently seen in the sample of a student's work shown below.

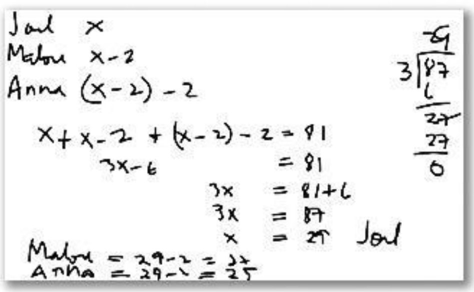

Figure 3. Sample work of global learner 2

However, only one reflective learner used algebra, specifically in answering problem 1. Performing calculations were also manifested by all reflective learners while applying heuristic approaches in solving the remaining problems that allowed them to specialize more by performing a lot of calculations in achieving reasonable answers. Moreover, like active learners, reflective learners reread problems while implementing the decided plan as revealed by the comment, *"Ang naaalala ko po, sa formula po ay age is equal to . . . Ay! Tama ba? Wait lang po."* (What I could recall is that, in the formula, age is equal to... Is it correct? Please wait.) [Reads the problem again.] Similarly, one global learner decided to use Algebra as she asked, "*Pwede po bang Algebra?" Pwede po si Joel ay x. Ang sabi po doon [problem] na 'two years younger' si Malou kay Joel, kaya minus 2 po sa age [of Malou].* (Can I use Algebra? Joel may be represented by x, Malou is two years younger than Joel, so we subtract 2 from the age of Malou) . Said global learner immediately proceeded to perform some calculations as she continued, "*Yung sum daw ng ages nila is 81 so magtranspose lang po then divide both sides sa 3."* (The sum of their ages is 81, so I transposed, then divide both sides by 3). Further, she reread some parts of the task to back up her solution. The solution of this global learner is shown in Figure 2 above.

In conjecturing and justifying, patterns were also discovered by reflective and sequential learners. In fact, a reflective learner said, "*May pattern po… kaya po, ididivide ko po sya* [referring to 81] *sa 3, 'tas ipa-plus 2 ko po. Kasi po tatlo po sila* [Anna, Malou, and Joel]." (There is a pattern, the reason why I will divide by 3, then I add 2 as there are three of them). Also, it was noticeable that the pattern observed by a reflective learner in solving a problem was similar to the pattern observed by two active learners. However, the reflective learner justified the observed pattern by performing calculations while highlighting the conditions in the problem as she emphasized, "*I find three numbers that summed up to 81 such that the one's places of the three numbers must sum up to 11 or 21. I observed that the ending of the sum must be 1* *because the one's digit of 81 is* 1." This "intelligent method" was also triangulated by her solution in another problem when she practically partitioned the principal amount (25 000) in the easiest way (e.g., first choice was 11 000 and 14 000; second choice was 10 000 and 15 000).

Moreover, all three global learners decided to use Algebra, but only one achieved the correct answer. This global learner asked, "*Pwede po bang Algebra?" Pwede po si Joel ay x. Ang sabi po doon [problem] na 'two years younger' si Malou kay Joel, kaya minus 2 po sa age [of Malou]*. (Can I use Algebra? Joel may be represented by x. Then, according to the problem, Malou is 2 years younger than Joel, so I subtracted 2 from that of Malou). Then, she immediately proceeded to perform some calculations as she continued, "*Yung sum daw ng ages nila is 81 so magtranspose lang po then divide both sides sa 3."* (The sum of their ages is 81, so I'll just transpose, then divide both sides by 3). Finally, she reread some parts of the task to back up her solution until she successfully completed the task.

On Looking Back and Generalizing

Notably, generalizing was not manifested by active, sequential, and global learners, that is, they neither generalized or even made a short statement about their answer nor did they looked back to check if their final answer was correct. The reflective learner who made the "intelligent method" as mentioned earlier was the only one who entered the *Review Phase* by summarizing the processes she took, justifying every step, going back to the problem, and providing short statements about her final answer. This is evidently seen in her solution in problem 5 as she wrote, "*You can buy 5 pens for P18.75 and buy 8 notebooks with P74. 00 and have remaining money of P7.25."* Further, she said during the interview, "*Tama po 'yong answer ko kasi po* *pag balikan po yong condition sa problem, e swak po… ang galing.*” (My answer is correct, because when I went back to look into the conditions given in the problem, it fits.... Very good!).

Pedagogical Implications

Understanding was visibly observed from all the learners as they entered the *Entry Phase* of the problem-solving process. By understanding the tasks, learners were able to highlight the key information to recognize the problems they had to deal with and to recall formulas and other algebraic solutions. Understanding the tasks was also the learners’ gate way to the *Attack Phase* for it allows them to think of a plan on how to solve the problem. Thus, the learners’ cognitive behaviors in performing problem-solving tasks may be enhanced by engaging them in activities that would develop and improve their skills of understanding the tasks, like asking them to rephrase the information in the problem and visualize the problem situation by using tables, models, and the like.

Specializing was the apparent cognitive process that all learners manifested to achieve answers to the problem solving tasks. Performing calculations was the main cognitive behavior manifested by all types of learners. However, sequential learners chiefly used algebra while specializing. Moreover, in *conjecturing*, the learners observed a pattern and used this to solve the pattern. The pattern was *justified* by performing calculations and monitoring the conditions given in the problem. These observations suggest that while setting up equations using symbols that are emphasized in Algebra is a strategy in problem solving, students may be taught how other strategies like tabulation, working backwards, discovering patterns, using blocks, and drawing figures may be used to solve problems.

Generalizing and *looking back* were not manifested by almost all learners. Though Mason, Burton, and Stacey (2010) emphasized the importance of generalizing and looking back as core cognitive processes in performing mathematical tasks, the findings of the study revealed that learners do not reflect on the reasonableness of their answer or at least be mindful to check whether their answer is correct or not based on their understanding of the given task/ problem. It is indeed a challenge to mathematics teachers how to encourage learners through problem solving to always look back, reflect on what was done to solve the problem, and try to extend/apply the ideas discovered to other problem situations because these are important processes done in making wise decisions.

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Appendix A. Sample Problem Solving Items

Below is the problem solving tasks used as instrument in the research paper "Learning Styles and Cognitive Processes of Students in Performing Problem Solving Tasks in Mathematics." Note that the problems are written on separate sheets to allow space for students' solutions.

PROBLEM SOLVING TASKS

Directions: Please read the problems below and solve them. Use the space provided for each item for your solution. You can select which item you want to solve first. You will also have to dictate your solution as you solve.

1. Anna is two years younger than Malou, and Malou is two years younger than Joel. If the sum of their present ages is 81, what would their ages be after three years?

2. Two printers can print 1000 certificates in 10 minutes. If one printer is used, printing 1000 certificates takes 30 minutes. How long does it take for the other printer to print 1000 certificates when it is used alone?

3. A 500-milliliter solution is 20% acid. How much water should be added to the mixture to get a solution that is 5% acid?

4. Mr. Santos invested a certain portion of P25000.00 for two years in a bank that offered a simple annual interest rate of 2%. He invested the remaining amount in another bank for one year with an annual interest rate of 3%. How much did he invest in each bank if he got a total of P900.00 interest from the two investments?

5. A pen costs P3.75, and a notebook costs P9.25. If you want to buy 5 pens, how many notebooks can you buy without exceeding your budget of P100.00?