Teaching Abstract Algebra Online

Erminda C. Fortes Philippine Normal University

fortes.ec@pnu.edu.ph

This research attempted to document the instructional plan that a professor would have to apply in teaching Abstract Algebra online using the descriptive case study design to come up with a framework on online abstract algebra teaching. The instructional preparations employed enabled the researcher to determine: the students' background knowledge of the content; the students' technical knowhow; and the resources available in the online environment. The Philippine Normal University - Learning Management System (PNU -LMS), Smart Notebook and Microsoft Power Point slides, and verbal discussions recorded on Techsmith Camtasia Studio, were used to create interactive asynchronous discussions. The students' outputs were used to provide immediate feedback regarding the developed materials and the discussions thereof. The following conclusions were derived: 1) Preparing for an online environment requires: planning the content; determining the nature of the students; determining the students' background knowledge of content and their technical know-how; and determining the quality of available resources; 2) The capacity of asynchronous discussions could be enhanced by giving opportunities for interaction between student and professor, student to student, and student to content; and 3) Assessing students' performance during and after the discussions, giving regular feedback, and eliciting students' feedback on the accessibility of the online class are some assessment techniques that an online instructor may use. Consequently, these conclusions served as basis for deriving a framework on online abstract algebra teaching. It was recommended that an experiment

on the effectiveness of the online teaching format in Abstract Algebra using the aforesaid framework be done.

Keywords: instructional preparations, online learning environment, self-directed learning

Introduction

Innovations in educational technology and the 21st century learners, also called "digital learners", are now of great influence to our educational landscape. Digital learners are those types of learners who are growing up in an advanced information technology environment. Much of the modern technology can be easily accessed by learners while electronic devices such as laptops and notebooks are drastically changing the way these learners think and learn. Shifting from face-toface to online learning may be considered as a way to match those digital learners who move constantly in an environment with progressive and rapidly changing innovations in technology. Online learning is popularly called e-learning. Horton (2006) defined e-learning as "the use of information and computer technologies to create learning experiences". On the other hand, Clark and Mayer (2008) claimed e-learning to be a form of training conveyed using a computer that is designed to reinforce learning of an individual.

There are many benefits of online delivery. In online formats, teaching and learning can occur at times that better suit students and teachers where both teaching and learning can take place almost anywhere depending on the availability of resources of both teachers and students. With the advent of technology, students do not depend solely on the teacher for their knowledge, unlike in a face-to-face class where the teacher has the control of the delivery of information (Instructional Strategies for Online Courses, n.d.). Thus, with the ideas of having digital learners and using online delivery in teaching, educators contemplate on the challenges to innovate and use technology-enhanced delivery in teaching. At the micro level, this task is really challenging to mathematics educators as well.

Mathematics is one of the courses where students can use varied educational technologies. Many students claim that they have to exert more time and effort in order to survive mathematics classes, which includes Abstract Algebra. Researchers believe that a solution to this is teaching the course complemented with the new advent of technology. In fact, the National Council of Teachers of Mathematics (NCTM, 2000) includes technology in its list of six principles that define the quality of teaching and learning mathematics. These principles point out that technology influences mathematics teaching and enhances students' learning.

Apparently, in response to the Philippine government's call for the country to be positioned in the global arena, efforts are done by educational leaders to cope with the high demands of integrating technology in teaching. The Commission on Higher Education (CHED), the Department of Education (DepEd), State Universities and Colleges (SUCs), and the academic institutions in collaboration with the nongovernment organizations (NGOs) and private organizations are trying to install high technology multimedia equipment in classrooms, and thus, some colleges and universities are now able to offer some of their courses online in order to cater to both local and foreign students. Consequently, most teachers and students all over the country slowly improve to have access to the varied state-of-the-art technologies and an opportunity to collaborate with other students around the world in a learning environment. This is one of the demands of learning in the 21st century.

To meet the demands of the 21st century, the country's National Center for Teacher Education, is mandated to provide support to continuing professional education of teachers and introduce technological and pedagogical innovations. Consequently, it offers graduate programs for teachers all over the country, but most graduate courses are offered on Saturdays because teachers have their teaching loads in basic education or tertiary level institutions on weekdays. Thus, the alarming issue of continuing decline in enrolment in the graduate programs yearly, which may be due to the following reasons: 1) overloading of teachers due to the K-12 curriculum; 2) teacher training and seminars that are mostly conducted on Saturdays; and 3) occasional cancellation of classes in basic education institutions, makeup classes scheduled on Saturdays. Online teaching may be the best way to address this decline in enrolment in the graduate program. Students in an online class may study based on their availability. Hence, if students enroll in online courses, they need not go to the university just to attend classes every Saturday.

Pressed with the challenges of creating methodologies that meet the demands of the fast advent of educational technology in the 21st century, introducing innovations in teaching mathematics and resolving the continuous shrink in enrolment yearly, the College of Flexible Learning in collaboration with the College of Graduate Studies and Teacher Education Research of the university initiated offering online courses in the Master of Arts in Education with specialization in Mathematics Education. Two math courses were offered online: Advanced Modern Geometry and Abstract Algebra. However, very few students enrolled in these online courses which were both handled by the researcher. This may be attributed to the fact that it was the first time that the university offered mathematics courses online which made students anxious perhaps in enrolling. Similarly, the researcher found it difficult to prepare handouts, lecture notes, motivation tools, discussion tools, and assessment tools that would be effective in teaching online.

In light of the above, the researcher was motivated to undertake a case study on teaching two important topics in Abstract Algebra – groups and subgroups, online. It was hoped that the case study would be able to document what an online professor has to do, from instructional preparations, delivery, to assessment, with meeting the demands of the new era of technology and ensuring the achievement of course objectives in mind. The document could serve as a reference for those who may want to design an online environment in teaching mathematics as offering online courses may be a solution to address the critical problem of deteriorating number of enrollees year by year. Finally, the findings of this study may serve as a model to introduce some technologyenhanced innovations in teaching that could build up students' learning of mathematics in the 21st century.

Literature Review

Instructional Preparations

Planning, developing, and organizing instruction are major parts of any teacher's job (Kelly, 2002). In order to properly plan and prepare for online instruction, a teacher must consider the following: the content to teach; students' characteristics and needs; available resources; available technology; and variety of instructional and assessment strategies (Planning and Preparing for Instruction: Standard 1). Other than preparing for the content to teach in an online class, a teacher needs to be familiar with the students' characteristics and needs. An online instructor must also determine available resources (print and online) that are suitable to the students and identify the technical preparations needed for both the teacher and students.

Delivery Modes

Educators should implement instructional strategies that are most effective for accomplishing a particular educational objective. There are instructional strategies which have been effectively used in a traditional classroom which can likewise be used in an online learning environment. Among these are the following: learning contracts, discussion, lecture, self-directed learning, mentorship, small group work, projects, collaborative learning, case study, and forum (Instructional Strategies for Online Courses). The use of these instructional strategies in an online class was presented by Pelz (2004) in his three "Principles of Effective Online Pedagogy". In the discussion of each principle, he mentioned specific techniques that he found effective in teaching his online classes. His first principle is - "Let the students do (most of) the work." His second principle, "Interactivity is the heart and soul of effective asynchronous learning," emphasizes that any course can be designed to make the role of the professor limited to providing the necessary structure and directions, supportive and corrective feedback, and evaluation of students' output. Lastly, his third principle: "Strive for presence," enumerates three categories of responses that add value to a discussion namely, Social Presence, Cognitive Presence, and Teaching Presence. In each category, he mentioned how a discussion in an online class may be made meaningful and valuable through the collaborative efforts of a professor and his students.

Assessment Techniques

During and after the delivery of instruction, the following best practices in assessment, among others, may be used: 1) The teacher aligns curricular goals with a variety of assessment tasks that reflect various types of knowledge and cognitive dimensions; 2) The teacher uses information

from individual and group assessment tasks; 3) The teacher provides high quality, timely feedback derived from informal and formal assessments; 4) The teacher uses assessment strategies that involves learners in self-assessment activities; and 5) The teacher uses a variety of test formats – some of which incorporate rubrics – across a unit of study to evaluate student understanding across the knowledge and cognitive process dimensions (Designing and Using a Variety of Assessments: Standard 2).

Purpose of the Study

This research was conducted to document the instructional plan that a professor would have to use in order to teach Abstract Algebra to graduate students online. Specifically, it aimed to:

- i. Determine the preparations that a professor has to do, discuss delivery modes of instruction that can be applied, and enumerate some assessment techniques that can be used in teaching Abstract Algebra to graduate students online; and
- ii. Develop a framework that can serve as a guide in teaching Abstract Algebra online.

Framework of the Study

Constructivism and Cognitive Theory

Stavredes (2011) cited that Cognitive Theory describes learning as a process in which learners construct their own understanding of the content by integrating prior knowledge, beliefs, and experiences. In a constructivist learning environment, the teachers play the role of a guide and help students connect their prior knowledge to new information. The researcher considered this theory in planning the preparations in designing the online environment. It was ensured that the students' prior knowledge was considered in said preparations.

Andragogy and Self-directed Learning.

The most familiar theory on adulthood is Knowles' andragogy. Knowles (1998) enumerated the characteristics of adult learners as follows: is self-directed, is goal-oriented, applies knowledge and experiences, relates the tasks assigned to them to their own goals to learn, implements theories in real life situations, and works collaboratively with others. As mentioned by Grow (1996), self-directed learning describes a process in which individuals take the initiative in diagnosing their learning needs, formulating learning goals, identifying human and material resources for learning, choosing and implementing appropriate learning strategies, and evaluating learning outcomes. These theories enlightened the researcher as to the kind of participant-learners needed in this study, and the insight served as a guide in determining the type of activities that could be effectively used with them. Consequently, the researcher prepared activities that would motivate them to diagnose their learning needs, decide on the learning resources that they can use, and assess their own learning. Further, the activities were chosen in such a way that students would be able to apply the learned concepts and the situations they experienced. Also, there were course requirements that compelled students to work collaboratively with one another.

Direct Instruction Model.

According to Pelz (2004), one way to add teaching presence in online discussions is by direct-instruction. Eggen and Kauchak (2001) noted that the Direct-Instruction Model is a teacher-centered strategy that uses teacher discussion including modelling coupled with allowing students to practice on their own and giving feedback. One distinguishing characteristics of this model is the interaction between the teacher and students. This theory was considered by the researcher in preparing the videos in which the concepts involved in the study are discussed. Consequently, in said videos, discussions were always complemented by activities where students could practice on their own. Further, students' feedback on the discussions and their experiences were always elicited, and the professor gave regular feedback as well.

In view of the above discussions, the researcher conceptualized a paradigm as shown below.

Figure 1. Graphical Representation of the Research Framework



The paradigm shows that three principles/theories were considered as theoretical bases in teaching Abstract Algebra online. Andragogy and Self-Directing Learning Theories were considered in preparing for and assessing instruction while Constructivism and Cognitive Theory, together with the Direct-Instruction Model, were taken into account in the delivery of and assessing instruction. Consequently, a framework on teaching Abstract Algebra online focused on preparing, delivering, and assessing instruction was derived.

Methodology

Research Design

This research used the descriptive case study design. It described in detail the preparations that an online professor has to make prior to students' exposure to the online environment, how to best deliver instructions, and assessment techniques that can be used to ensuring attainment of course objectives.

Participants

A class of 20 students in Math 503 (Abstract Algebra) pursuing Master of Arts in Education with specialization in Mathematics Education participated in the study. The class was scheduled to meet during the second semester of SY 2014-2015 for 18 consecutive Saturdays. Usually, the class sessions were purely face-to-face. However, in this course, six sessions were delivered online while the remaining sessions, face-to-face. Further, only the six sessions that were delivered online were considered in this study.

Research Instruments

The researcher made use of a 10-item pre-test and a 15-item Likert-type questionnaire to determine the students' knowledge of the topics and technical know-how prior to the online delivery. Further, interview guides, rubrics

to assess the students' outputs, and portfolios of student experiences were used during the online delivery. Also, the book *Contemporary Abstract Algebra (5th ed)* by Joseph A. Gallian and a course pack written by the researcher were used as primary references for the course. Finally, a 10-item post-test to determine what the students learned and a 20item Likert-type questionnaire to assess their perceptions of the teaching-learning process in terms of instructional preparations, delivery modes, and assessment techniques were administered after the online delivery.

Data Gathering and Analysis

The class was oriented on how Math 503 (Abstract Algebra) would be delivered during the 2nd semester of SY 2014-2015. It was made very clear to the students that the first part of the syllabus, a discussion on Preliminary Concepts, would be delivered face-to-face, the middle part that covers Groups and Subgroups would be delivered online, and the remaining part will again be face-to-face. Also, it was explained that the online delivery would be considered a case study, and the face-to-face treated as usual sessions. Furthermore, during the orientation, the details of the course and the syllabus were discussed.

The face-to-face delivery of the first part of the syllabus, Preliminary Concepts, lasted for three sessions. This activity was complemented by individual student interviews. Both activities informed the researcher about the nature and characteristics of the students as adult learners. The face-to-face syllabus delivery and individual student interview activities were used as base in determining the type of support that the researcher would extend to the students and the most effective modes of instruction delivery to could be used.

Instructional materials for the online topics were prepared prior to the online sessions. These

included handouts, powerpoint presentations, and video presentations. Lectures were prepared using Smart Notebook slides and the researcher recorded the lectures using Techsmith Camtasia Studio. Similarly, the instruments to assess the online learning environment were developed and content validated by experts.

The technical preparation of the students was done in one session. This involved getting familiar with the Learning Management System. Thereafter, the two topics (Groups and Subgroups) were delivered for six sessions using said system. In the six session instructions, the researcher played the dual roles of a course professor and an educational researcher. The learning and assessment tools previously prepared by the researcher were utilized during the teachinglearning process but were also continually improved while instructions were on-going. This was done to ensure that the performance and learning of the students were fully monitored and maximized. Finally, during and after the sixsession online instruction, data werecollected and analyzed.

Frequency counts, percentages, and means were computed to describe the participants' knowledge of the topics and technical know-how before their exposure to the online environment, what they learned after the said exposure, and their perceptions of the preparations, instructional materials, modes of instruction delivery, and assessment strategies that the researcher implemented during the online sessions. Results of the interviews were used to triangulate and substantiate the quantitative data obtained.

Results and Discussion

Instructional Preparations

As seen from the literature, it is indeed necessary to consider the learners when designing the content of instruction and its delivery whether in a traditional classroom or in an online environment. Observations during the face-to face delivery and the preliminary interview with the students served as bases for the researcher to determine the nature of learners that would be involved in the online class. A summary of the result is presented below.

Nature of Respondents	Frequency (n = 20)
involved learners	16
collaborative learners	12
self-directed learners	12
interested learners	4
familiar with the concepts of groups and subgroups	4
knowledgeable of the theorems and tests for subgroups to use	1
know how to convert Microsoft Word files into PDF formats	10
using resources from the internet to browse the topics in their math	20
courses	
using resources from the internet to enhance learning	12
find browsing the internet time consuming	10
with e-mail accounts	20
with g-mail accounts	6
had experienced using the LMS	0

 Table 1.
 Nature/Description of Learners

It may be gleaned from Table 1 that majority of the students were "involved, collaborative, and self-directed learners". Involved learners, as described by Grow (1996), have skills in and knowledge of the subject, and they have a sense of where they are going and how the course fits with their goals. They feel confident and motivated that they can get there, and they respond to an instructor who will help facilitate their progress through unfamiliar content and offer appropriate tools, methods, and techniques. Moreover, collaborative learners, as described by Grasha (1996), learn by sharing and cooperating with the instructor and other learners. This result guided the researcher on the kind of activities that would be used in the delivery of the online instruction. Consequently, activities that would maximize student-teacher and student-student interaction were prepared. These included the following: 1) problems/ exercises that would be solved/done with the guidance of the professor and eventually on their or with their group mates; 2) asking questions that would enable students to react to the professor; and 3) citing comments that would prompt students to ask questions or convey their difficulties at any time.

However, 20% of the participants were identified as "interested learners". Interested learners, as depicted by Grow (1996), have little or no prior knowledge, but they are interested in learning and respond to instructors who are motivators. This result prompted the researcher to confirm the students' background knowledge of content before their exposure to the online environment. It was then revealed that only 20% were familiar with the concepts, and 5% were knowledgeable of the principles that are pre-requisite to the course. Consequently, lectures to be recorded were carefully planned to cater to those with little prior knowledge and ensure that they would be motivated to learn, initially with the guidance of the professor and eventually on their own. Also, though the book entitled "Contemporary Abstract Algebra (5th ed)" written by Joseph A. Gallian contains all topics that would be discussed, the researcher decided to write a coursepack designed for the type of students as revealed in Table 1 and to supplement the verbal discussions. A copy of the coursepack was provided to the students during the first session

Further, a needs assessment of the students in terms of their technical know-how was done. Table 1 revealed that not all students were familiar with the technological pre-requisites of the online environment. Consequently, the researcher was prompted to do the following prior to the online delivery: 1) train the students to convert Microsoft Word files into PDF formats; 2) require 70% of the students to open g-mail accounts where videos would be sent; 3) allot one session to acquaint the students with the PNU LMS and give them a week to explore said platform; and 4) browse and evaluate other print and online resources that could enhance students' learning and require students to access the online references.

Finally, students were asked to assess the instructional preparations done by the researcher. The mean ratings of the scores were interpreted as follows:

Mean	Interpretation	
1.0 - 1.99	fair	
2.0 – 2.99	satisfactory	
3.0 – 3.99	very satisfactory	
4.0 - 5.0	excellent	

Table 2. Mean Ratings and Their Interpretation

The item "*The course syllabus clearly outlined the part for the online environment.*" received a mean rating of 4.45. This indicates that students clearly found the delineation between the face-to-face and online environment in the syllabus. Further, the item "*The online class provided a variety of resources for the content.*" got a mean rating of 4.85 and was supported by the comment given by student 17 as follows: "*It is good that you gave a list of numerous references, both print and online, that I could use to understand the content better.*" Moreover, it is very evident that students found all web resources available and useful throughout the

online environment as the item "All web resources were visually and functionally consistent throughout the online class." received a mean rating of 4.20. This is supported by the following comments of students on the web resources: S10) "I was able to read more about the topics that you discussed because you gave us many online references that I found useful."; and S15) "Thank you for providing us a lot of resources that I found useful in studying all topics that we discussed." Finally, the two other statements that obtained mean ratings with interpretation of "excellent" are as follows: "The online environment had numerous activities that are appropriate to assess student readiness for the course." (with rating of 4.50); and "A wide range of resources is accessible that reinforced/enhanced the content and various learning abilities of students." (with rating of 4.40).

Delivery Modes

According to Pelz (2004), one way to add teaching presence in asynchronous discussions is by direct-instruction. In the Direct-Instruction Model, the teacher discusses the content, models the procedures involved, guides students in their practice, allows them to work on their own, and gives feedback. Consequently, the Smart Notebook was used by the researcher during the asynchronous discussions by directinstruction. Smart Notebook is a learning software that is built for education. It contains slides that are similar to power point slides, and the toolbar on each slide enabled the researcher to write the definitions, theorems and their proofs, solve problems that illustrate a theorem/principle, highlight important points, and summarize the discussions. Below is a screenshot of a Smart Notebook slide that the researcher used in the online environment.



Figure 2. Screenshot of a Slide of Smart Notebook

Furthermore, to complement the Smart Notebook slides with verbal discussions in a fashion similar to that when using a chalkboard or a whiteboard in an ordinary classroom, the researcher posted some questions while discussing the lesson verbally. The slides in the Smart Notebook and the lecture discussions were recorded using Techsmith Camtasia Studio. Following are screenshots of a frame during and after recording.



Figure 3. Screenshot of a Frame During Recording



Figure 4. Screenshot of a Recorded Frame

In order to maximize the benefits of the lecturediscussion, Microsoft Power Point was also used to enable the students to write their inputs on the text. Said inputs may be in the form of a question for further clarification, answers to the questions that were embedded in the discussions, proofs of theorems or statements/reasons that are missing in a proof, solutions to the problems posted, and answers to the exercises that were provided after the summary of the discussions.

Slides in Microsoft Power Point and the videos recorded in Techsmith Camtasia Studio that were converted into MP4 were uploaded through students' g-mail accounts. The researcher posted announcements, individual and group tasks, and their deadlines for submission while the students uploaded their files in the "news forum" of the PNU LMS.

Moreover, after discussing the content online, students were asked to evaluate the different delivery modes employed by the researcher with the use of the assessment instrument. Following are the items with the corresponding computed mean rating.

Table 5. Students Lyaluation of Denvery Modes	Table 3.	Students'	Evaluation	of Delivery	Modes
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Item	Mean Rating
 Students are provided sufficient opportunities to communicate and interact to the content, other students and the professor. 	4.85
 Learning objectives are achieved and learning activities are clearly integrated. 	4.20
 The online class provides varied forms of activities (textual, visual and auditory) that reinforce/enhance learning of students. 	4.65
 The online environment provides several activities that train problem-solving and critical thinking skills of students. 	4.60
 The online class uses multiple technology tools that facilitate learning and communication. 	4.10
 The innovative teaching methods used by the professor are engaging and enhanced student learning. 	4.80
 Different learning styles of students are considered with the use of a variety of useful multimedia resources during the entire online class. 	4.15
8. The online environment effectively engages students in the learning process.	4.85

The quantitative data above revealed that the delivery modes utilized in the online environment are varied, engaging, and are effective means to enhance learning and develop various skills of students. This was reinforced by the students' comments that they mentioned either during interview or on their portfolios. Said comments revealed that the video-lecture of the professor gave them activities that would enable them to solve problems while letting them participate in the discussion board allowed them to think critically. As emphasized by Synder and Synder (2008), students' critical thinking skills could be enhanced by using instructional strategies that actively engaged students in

the learning process. Following are examples of the abovementioned comments that served as evidence for the mean rating of 4.6 for item 4 (*The online environment provides several activities that train problem-solving and critical thinking skills of students.*) and the mean rating of 4.8 for item 6 (*The innovative teaching methods used by the professor are engaging and enhanced student learning.*).

"It's good that the professor gave several examples of groups and subgroups, so when I was asked to choose which of the given sets are groups, I found it easy already. I also enjoyed answering the questions that you asked in the video. I was trained how to speak my mind in written form. Now I know how to prove; at first, I found it difficult, but with your encouragements, I know I have improved a lot."

"I really learned when you asked us to post our discussions in the discussion thread. My classmates are also a good source of information. Also, I learned how to critique the answers of my classmates, and my misconceptions were also corrected most of the time."

Moreover, the comments cited above further disclosed that students were exposed to activities that allowed them to interact not only with their professor but also with the content and to their fellow students as well. Thus, these comments also support the mean rating of 4.85 for item 1 (*Students are provided sufficient opportunities to communicate and interact with the content, other students and the professor*.).

Assessment Techniques

In discussing Groups and Subgroups online, students were required to interact with the professor, with the content and problems discussed, and with each other, either in pairs, small groups, or entire class. Further, students were allowed to interact in text and verbal discussions, problem sets, and illustrative examples. Individual inputs were written on Microsoft Power Point slides, and outputs were sent in PDF format. On the other hand, collaborative reactions were written on threaded discussion boards in the PNU LMS.

Each student was required to interact with the text in Microsoft Power Point slides and with the researcher's verbal discussions. Inputs were assessed by the researcher with the use of an input rating scale.

Included in the individual tasks of each student is to find a website that discusses information related to a content discussed in the online environment. A student is required to discuss the highlights of the discussions on the website and write his/her reflections on the discussions. Outputs of students were submitted in PDF format and were rated by the researcher based on the quality of both the website and analysis done.

Two problem sets were assigned that required each student to apply the definitions and principles discussed. To assess the outputs of students, the pairs check strategy was used by the researcher wherein a partner was assigned to each student, and the partner's task was to check his/her partner's solution using the answer key provided by the researcher. Similarly, two problem sets were assigned for students to accomplish collaboratively. For the first problem set, students selected their own group mates to work with. On the other hand, the researcher determined the composition of each group for the second problem set based on their academic ability, as revealed by their individual outputs. The researcher required each member to share his/her ideas with the group on how to accomplish each problem set and talk about them on a threaded discussion board. A rubric was provided for each member in order to assess his/her own and other members' contribution in accomplishing a task. Likewise, the

researcher assessed the performance of each student based on the outputs written on the discussion board.

The researcher gave deadlines for submission to help students estimate how long they were expected to finish doing a task. This was complemented with text messages sent to them as a reminder when deadlines were coming up.

Feedback was given by the researcher to each individual or for each group task. Written below are transcripts from student interviews and portfolios on the benefits that students gained from feedback:

"When the professor gave feedback on my proofs in the Exercises, I was clarified of the errors that I made. She pointed out why an error was committed and what should be done instead." So I did not commit the error in the Problem Set. I also learned from the feedback given to us when we submit group works."

"I was inspired when I received positive feedbacks, but I also learned from negative feedbacks, I mean, comments why my answer was wrong.

"Hurray to my professor! She always gives feedback right after my answers and solutions were submitted. I learned so much from the feedback. I hope to have the same experience even with my ordinary classes, I mean not online classes."

"The feedback given to our group made us realize that indeed, comments coming from teachers are necessary to motivate students, as we felt this way whenever we receive feedback from our professor".

The above comments can be considered an evidence for the mean rating of 4.6 for item 4 on assessment techniques

as presented in Table 4 below.

Table 4. Students' Evaluation of Assessment Techniques

	Item	Mean Rating
1. ⁻	The course objectives, instructions, and assessment activities are closely aligned.	4.20
2.	The instructor offers varied opportunities for students to give feedback about the content and monitor their own performance/ progress.	4.70
3.	There is a variety of assessment tools that are used to measure students' knowledge and skills.	4.15
4. 3	Students' peer and self-assessments are considered throughout the online class.	4.75
5.	Timely and regular feedback about student performance is provided throughout the online class.	4.60

Finally, the result of the post-test administered at the end of the online environment disclosed that the 20% of students who were already knowledgeable of the concepts of groups and subgroups as revealed by their pretest significantly improved after their exposure to the online environment. Initially, these students hardly identified which of the given structures form a group/subgroup. In fact, no one was able to fully justify the corresponding answers. Fortunately, 100% of those students succeeded in identifying groups and subgroups from the given structures and in justifying their answers. Similarly, the remaining 80% of the class were found to be knowledgeable of the concepts of groups and subgroups after their exposure to the online environment. Furthermore, of the 20 students, 17 learned how to apply the different procedures, theorems, and tests in proving group and subgroup structures. It may be noted that the course objectives include the following: define groups and subgroups; cite examples of groups/subgroups and their properties, as well as justify why they are considered groups/

subgroups; and prove the theorems and principles pertaining to groups and subgroups. Evidently, the results of the posttest are indicative that the course objectives were achieved after exposing students to the online environment. This claim and the mean ratings for the items on assessment techniques were supported by the student interviews and the experiences that they wrote in their portfolios, some of which are quoted below:

"Thank you for allowing us to ask questions regarding the topics that we did not understand well. I learned that we can also require our students to ask questions and write them in Word file or in power point presentations. I would want to try it with my class, as I found it effective in our online sessions."

"I enjoyed doing the Problem Sets with my group mates. Now, I know how to prove. Before, I hate giving Problem Set as a group activity because I know my students will just copy from intelligent ones. But with my experience in this course, now I know how to use the strategy, and I now love it, so I will do the same with my students."

"I learned from you and my group members. I am used to doing my tasks alone, so I did well in the individual tasks, but when we did a task as a group, I also learned. I could speak freely, and it made me more confident that my thoughts are right because my group members affirmed my answers, and so by you, Ma'am. At first, I thought I could do better when I do it alone, but I was wrong. Really, two heads are better than one. All members of our group were cooperative; every one finds time to do the assigned tasks. I also like the experience when every member of the group gave feedback on others' outputs." "At first, I found it difficult to discuss a topic that I learned from a website. But when I was about to complete the task, I enjoyed it already. I felt proud that I can do it, that I can explain what I learned from a source because it affirms that I really learned something on my own."

"The rubrics given to us guide us how we will be graded, so it helped us a lot. I am also using rubrics with my class, and I thought I can just use it in group works. Now I learned that I can use it even with individual outputs."

Framework in Teaching Abstract Algebra Online

Just like teaching in an ordinary classroom, teaching Abstract Algebra online involved three phases: preparing for instruction, delivering instruction, and assessing instruction. However, other than the preparations that are usually done in a classroom set-up, like preparing for the content to teach and determining the nature of the students as well as their background knowledge of the content, a professor should also consider the quality online resources that students can use to enhance their learning in an online environment. Also, the technical know-how of the students should be assessed. and the necessary training should be provided prior to their exposure to the online environment. Moreover, delivering instructions can be enhanced when students are exposed to activities that give them opportunities to interact with the content, with each other, and with their professor. These may be done using an appropriate Learning Management System that both the professor and students are familiar with. Lastly, assessing instruction can be done more effectively by eliciting students' feedback not only after the delivery of instruction but while instruction is being delivered as well. This should be coupled with the professor's regular feedback

on the student's performance. Drawing out feedback from students would give the professor an opportunity to decide whether to continue or modify the planned instruction for better results. Further, professor's regular feedback gives students an impression that the professor is with them in their entire journey. While ideally these should also be done in a traditional classroom, these are essential in an online environment to enable students manage their learning experiences.

Conclusions and Recommendations

Abstract Algebra is one of the courses that most students find difficult to learn, understand, and appreciate. However, with the advent of technology, this notion may be completely revolutionized by teaching it online. In doing so, careful steps on instructional preparations, delivering the content, and assessing if the objectives are achieved should be done. The results of this research served as a basis for the researcher to conclude the following: 1) When preparing for an online environment, an instructor should take into consideration the content to teach, the nature of the students and their background knowledge of the content, an assessment of the quality of available resources (print and online), the technical know-how of the students; 2) The capacity of asynchronous discussions can be enhanced by giving opportunities for interaction between student and professor, student to student, and student to content. 3) Assessing students' performance during and after the discussions, giving regular feedback, and eliciting students' feedback on the accessibility of the online class are some assessment techniques that an online instructor may use. These conclusions may be considered by math educators in deriving a framework on online Abstract Algebra teaching. However, it may be noted that this research did not use an experimental design that would look into the effectiveness of teaching Abstract Algebra online using the derived framework. Consequently, it is hereby recommended that an experiment on the effectiveness of the use of an online format in teaching Abstract Algebra using the aforesaid framework on instructional preparations, delivery modes and assessment be done.

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