Mathematical Creativity in Solving Non-Routine Problems

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Abstract The study used the descriptive design to explore the students' mathematical creativity in terms of fluency, flexibility and originality in solving six non-routine problems. Thirty (30) participants chosen using stratified sampling from 123 Grade 10 students, were asked to solve six non-routine problems. In each of the three sessions, two problems were answered by them, after which they were asked to write a journal about their experiences in solving a problem and then they were interviewed. Solutions of the participants which succeeded by an interview that were interpreted using validated rubrics described their mathematical creativity in terms of fluency, flexibility and originality. Results show that students may be described as "moderately creative" in all three components fluency, flexibility, originality. Likewise, the over-all level of mathematical creativity in solving non-routine problems is *moderately creative*. Consequently, the researchers derived pedagogical implications to improve the mathematical creativity of students.

Keywords: flexibility, fluency, mathematical creativity, and originality

Introduction

The K to 12 Curriculum has been the basic education curriculum of the Philippines since the school year 2011 to 2012. One of the

Suggested Citation: Fortes, E.C., & Andrade, R.R. (2019). Mathematical Creativity in Solving Non-Routine Problems. *The Normal Lights*, 13(1), 108 – 135.

major goals of the curriculum (perhaps what DepEd Secretary Bro. Armin Luis Luistro refers to as “a change in perspective”) is making education truly relevant to the learners by using materials and techniques that are applicable to the learners’ daily lives. In particular, critical thinking and problem solving are now regarded as the twin goals of mathematics teaching (K to 12 Curriculum Guide, 2012, p. 3). This is a deviation from the common perspective that mathematics is all about memorizing formulas and solving equations.

Relevant to such change, Piggott (2011) claim that a curriculum that encourages problem solving needs to provide independence by empowering learners to think by themselves; teachers should support interactive environment by giving the students the opportunity to develop a habit of the mind that lead to meaningful experience. One of the countries that have successfully executed this scheme is Singapore (Clark, 2009). Consequently for the past ten decades, Singapore has been one of the best performing countries in mathematics education, as shown in the Trends in International Math and Science Study (TIMSS) comparison assessments in 1995, 1999, and 2003; it even reached top three in 2007 (Clark, 2009). The main reason for this achievement of Singapore is the priority now given to problem-solving, described to be the core of mathematics learning. It involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real world problems (Clark, 2009). This point of view may be a step closer towards the goal of improving the general performance of Filipino students whose mastery level in Mathematics is constantly decreasing. Consequently, if Filipino students would be exposed to non-routine real world problems, then their understanding of mathematical ideas and algorithms could be deepened and extended, and thus, resulting to higher mastery level. Notably, as shown in the low results of the annual National Achievement Test and 41st in Math among 45 participating countries who joined the 2003 Trends in International Mathematics and Science Study (TIMSS) (Herrera, 2014). In addition, the poor performance of the Philippines in education is evident in TIMSS 2008 wherein Filipinos scored lower than many other countries, even ranking last in math (Cruz, 2010). Thus, the need to improve the mathematical skills of students is not only necessary but an urgent and gravely important task.

Apparently, in order to help Filipino pupils become flexible and creative problem solvers, teachers should give their students the opportunity and freedom to try their own original solutions (Borja, 2011). In fact, students become good problem-solvers if their analytical and critical thinking skills are enhanced in education (Vistro-Yu, 2012). Being a good problem solver is characterized not only by the ability to compute and perform mathematical procedures, but by the capacity to apply mathematical skills in other subjects and in everyday situations, which can be developed if they are exposed to non-routine problems. (Dendane, 2009)

Routine problems require direct application of algorithm(s) and are solved through the use of previously learned procedures (Mabilangan, 2011). Thus, strategies in solving these problems limit students learning by developing only memorization and execution skills. Comparatively, non-routine problems are challenging problems that encourage the use of different heuristics in solving problems and provide true learning opportunities for students (Dendane, 2009). A significant number of studies in mathematics education (Celebioglu, Yazgan,& Ezentas, 2010; Mabilangan, Limjap, & Belecina, 2011; Villareal, 2014; Yazgan, 2015) suggest that non-routine problems are the most effective in honing mathematical problem-solving skills. More importantly, the practice of solving non-routine problems increases the probability that students will use these skills in real-life situations.

Mathematical Creativity

Borja's (2011) study showed that among the components of mathematical creativity (fluency, flexibility and originality), students performed best in fluency and least in originality, implying that generating many correct solutions is the easiest and coming up with original solutions is the most difficult. However, this result is in contrast with what Leikin (2009) found that originality is the strongest component of mathematical creativity; therefore, efforts should be focused on the other areas of creativity. Furthermore, Kattuo, Kontoyianni, Pitta-Pantazi and Christou (2011) claimed that the average mathematical ability students have an average performance across fluency, flexibility and originality.

These above mentioned studies on mathematical creativity included solving non-routine problems. The results showed that the use of non-routine problems is most effective in honing the mathematical creativity of students. Other studies in solving non-routine problems presented varied results in terms of performance and creativity. For instance, Yazgal (2015) who analyzed the role of strategies in solving non-routine problems found out that the more successful in distinguishing students of high and low performance include Look for a Pattern and Guess and Check strategies. Moreover, the analysis of students' solutions conducted by Mabilangan, Limjap, and Belecina (2011) showed that when students are given the freedom to solve as they please, seven out of the eight problem-solving strategies were used at least once to solve non-routine problems.

To recap, studies have been conducted to promote the development of creative thinking of students. Results showed that exposing students to non-routine problems yields positive effect in the mathematical creativity of students. Consequently, this research aimed to contribute to such body of works by describing and comparing the mathematical creativity of students in terms of fluency, flexibility and originality. This also hopes to fill the gap in the current literature by involving students in a national high school in contrast to those who are studying in high-achieving schools covered in the previous studies in the country. In this way, the previously established trends would be verified.

This research aims to contribute to this growing body of works by measuring and comparing the mathematical creativity of Grade 10 students of a National High School in the country. These students belong to the first batch of the K to 12 curriculum, and therefore, could provide invaluable information and perspectives. While there is no universally accepted definition of mathematical creativity, Kontorovich and colleagues (2011) identified three components of mathematical creativity as fluency, flexibility and originality. Likewise, based on the review of literature, these three components of mathematical creativity are suggested and are utilized to measure the mathematical creativity. Fluency was defined as a person's ability to extract a large number of correct solutions. Flexibility was referred to a person's ability to shift from one way of thinking to another and extract solutions in different categories. Finally, originality was defined as the person's ability to approach the given problems in a new/unique way and extract unexpected and unconventional solutions.

Framework of the Study

The Triarchic Theory of Intelligence (Sternberg, 1997; Sternberg and Lubart, 2000) defines creativity as the ability to produce unexpected, original work that is useful and adaptive, and considers it as the central component of intelligent human behavior. Similarly, Kwon, Park, and Park (2006) proposed a definition of mathematical creativity as the creation of new knowledge and flexible problem solving abilities. Thus, creativity (Kumar, 2014) includes the following: (1) Ability to create new ideas, theories or objects; (2) Capacity to synthesize ideas and develop an unexpected original work; (3) Freedom to exercise choice; and (4) Skill to solve problems and come up with unique solutions. As previously stated, Kontorovich, and colleagues (2011) cited three components of mathematical creativity as fluency, flexibility and originality. It is in this premise that fluency, flexibility and originality in solving non-routine problems were the ones considered to measure the mathematical creativity of students as depicted in Figure 1. This afforementioned components are represented or described as (Kumar, n. d.): (1) Fluency- judged on the basis of the appropriateness of the response when considered in relation to the test problem; (2) Flexibility- represented by a person's ability to produce ideas which differ in approach or thought trend; and (3) Originality- represented by uncommonness of a given response.

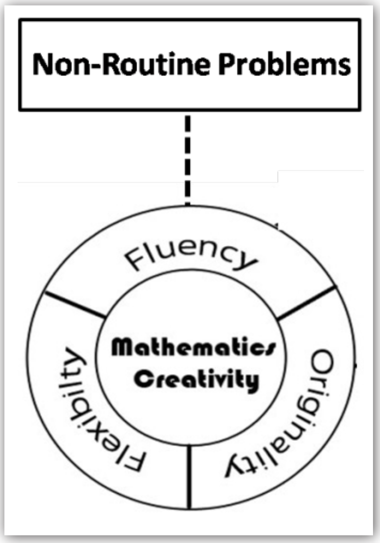


Figure 1. The Framework of the Study.

Purpose of the Research

The main purpose of this research is to describe the mathematical creativity of students in a National High School in Laguna. Specifically, the study sought to answer the following questions:

1. How mathematically creative are the individual students in terms of fluency, flexibility, and originality?
2. What is the group's overall level of mathematical creativity in solving non-routine problems?

Methodologies

Research Design

The descriptive design described and analysed the mathematical creativity of students in terms of fluency, flexibility and originality. It made use of both quantitative and qualitative data gathered from students' mathematical solutions, portfolios and interviews.

Participants/Respondents

The researchers used stratified random sampling to choose thirty (30) participants from one hundred twenty-three (123) Grade 10 students of a National High School. Each strata of mathematics achievement - above average, average, and below average, includes ten (10) students who were randomly selected using their averages in the their averages in the first two quarters.

Instruments Used

Survey Questionnaire

Part 1 of this instrument focuses on the students' personal information and educational background, while Part

2 focuses on their interest on Mathematics. Nine mathematics experts from two prominent teacher training universities of the country validated this questionnaire.

Journal Writing Notebook

Using a journal writing notebook, students were asked to reflect on their solutions to each problem. Likewise, they were provided guide questions to allow them to explain the strategies they used and reflect on their thoughts on the problem's level of difficulty. Two sample questions are: *How did you arrive at your answer in problem 2? Why did you do that?*

Interview Guide

This instrument was used with all thirty (30) students and the questions include an explanation on their first impression of the problem, identifying the problems that were most difficult for them and giving clarification for their solutions. A pair of sample questions is: *Which of the six problems did you find most difficult to solve? Why?*

Problem-solving Test

Students were given six non-routine problems (Appendix A) with the New Sourcebook for Teaching Reasoning and Problem-solving in Junior and Senior High School (Krulik and Rudnick, 1996), Problem-solving Heuristics of High School Seniors (Laset, 2003) and Metrobank MTAP-DepEdMath Challenge (MTAP, 2011; 2012; 2013; 2014; 2015) used as references.

A panel of experts from two teacher training universities evaluated thirteen non-routine problems and the six non-routine problems with the highest mean score were included in the Problem Solving Test.

Problem Solving Rubric and Scoring Rubric

Nine experts validated the problem solving and scoring rubrics (Appendix B). They are the same experts who validated the Problem Solving Test.

The problem-solving rubric developed by the Oregon Department of Education (1991) was adopted. A point system (proficient – 5 points, apprentice – 3 points and novice – 1 point) similar to the one used by Mabilangan, Limjap and Belecina (2011) and Villareal (2014) was the one used to evaluate the performance of the students in solving non-routine problems.

On the other hand, the scoring rubric used to determine to what extent solutions exhibited fluency, flexibility and originality had the following point system: high – 5 points, moderate – 3 points and low - 1 point. A score of 2 (or 4) was given to a work that would exceed the criteria for a score of 1 (or 3), but would not meet criteria for a score of 3 (or 5).

Data Collection and Analysis Procedures

The problem solving test was administered individually to 30 Grade 10 students. The six non-routine problems were given in three sessions. Students were interviewed individually for at least 30 minutes and the interviews were audio-taped. Likewise, they were asked individually to write a journal about their experiences in solving each problem.

While solving, students were allowed to ask questions about the direction and clarifications about the problem. Each session is equivalent to three hours. Three mathematics professors were oriented on scoring, after which they were individually asked to analyze and score the students' solutions using the scoring rubric. When greatly varied scores were initially given, they discussed until they arrived at a consensus.

The following mean scores were used to describe students' mathematical creativity.

Levels of Mathematical Creativity	Mean Intervals
Highly Creative	20.6 - 30
Moderately Creative	10.6 – 20.5
Lowly Creative	1.0 – 10.5

Ethical Consideration

Prior to the implementation of the data collection process, the researchers sought the consent of parents, teachers and students to participate in the study, particularly during test administration and interview (including recording and audio-taping). Likewise, the researchers ensured the safety of the participants during the test administration and interview. The participants did not receive any honorarium from the researchers but they were informed of the results of the study after its conduct.

Results and Discussions

Overall Mathematical Creativity

Table 1 below shows the students' average scores in each component of mathematical creativity when they are grouped according to level (high, moderate, low).

Table 1. Students' Average in Mathematical Creativity

Levels of Mathematical Creativity	Mean (Fluency)	Mean (Flexibility)	Mean (Originality)	Grand Mean
Highly Creative	24.21 (N = 14)	25.63 (N = 8)	25.00 (N = 3)	24.76 (N = 25)
Moderately Creative	17.13 (N = 15)	16.70 (N = 20)	14.46 (N = 13)	16.23 (N = 48)
Lowly Creative Students	10 (N = 1)	10 (N = 2)	7.21 (N = 14)	7.71 (N = 17)

Grand Mean	20.20 (N = 30)	18.63 (N = 30)	12.13 (N = 30)	16.99

As gleaned from Table 1,students' level of mathematical creativity in solving non-routine problems is moderately creative. This means that students were able to show correct solutions/answers and the strategy used is the one being used by 50 percent of students' population. However, students commit some errors in their solution/s though they used appropriate information and ideas.

Among the three components of creativity, students performed best in fluency. The majority obtained the answers correctly, but the students had difficulty in generating unique solutions, making originality as the least component of mathematical creativity (Borja, 2011). This may be attributed to the notion of students being comfortable when using only one strategy and did not think of alternative ways to get and verify their answers. Some of them even got an incorrect answer yielding no point in originality.

Mathematical creativity in terms of fluency, flexibility and originality

On Fluency

A total of 14 students are considered *most fluent*. These are the students who are able to give the correct answers and solutions while committing minimal errors or no errors at all. These students thoroughly investigated the situation and were able to verify their answers by applying all related information in solving the problem.

Notably, the most fluent among all the respondents proved to be very careful in performing the needed operations, even showing detailed explanations for each strategy. In solving the six problems, Student A shows fluency in solving the 2nd problem by using a table, chart and listing all possibilities. Problem 2 with the correct answer is presented below followed by the solution of Student A:

> **Problem 2:** A mathematics quiz consists of 50-multiple choice questions. A correct answer is awarded 5 marks and 2 marks are deducted for a wrong answer while no marks are awarded or deducted for each question left unanswered. If a boy scores 172 marks in the quiz, what is the greatest possible number of questions he answered correctly? Explain how you worked it out.

> **Correct Answer:** The greatest possible number of questions that the boy answered correctly is 38.

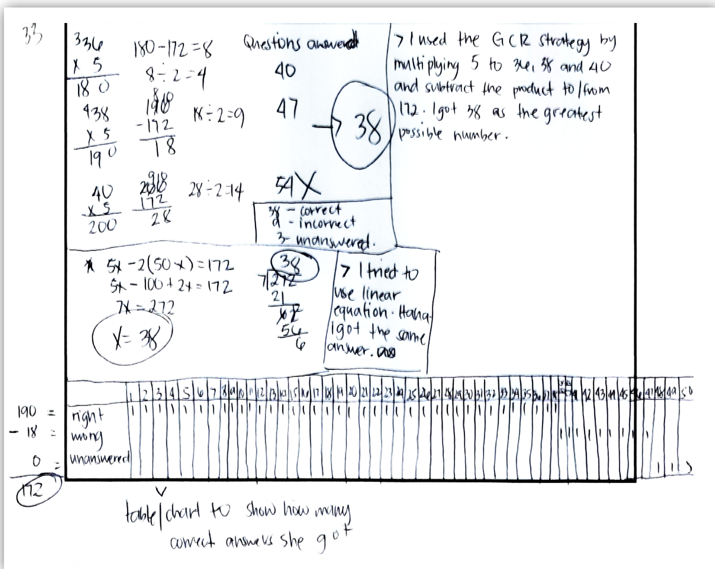


Figure 2. Solution of Student A in Problem 2.

Referring to Figure 2, Student A applies important concepts on the correct data, series of operations and trial-and-error computation. Furthermore, she explains her solutions by using tables, chart and writing down the possible answers for the 50 questions. In fact, to verify her answer, she uses a linear equation in one variable in solving the problem as shown in the encircled part "x = 38". The solution of Student A shows her ability to use several strategies without committing any error in her solution, and thus, considered as one of the most fluent students.

Students who earned a total score of 11-20 are considered "*fluent*". These students committed some errors in their solutions and got the correct answer in most problems, but did not succeed in the remaining problems. Some of them only guess the answers, while others use concepts that are actually insignificant. For instance, Student B correctly answered four questions but missed out the two remaining problems. Also, there were instances that Student B was able to get the correct answer but was unable to justify the answer. Based on his journal and interview, he was trying to use some concepts which are not directly stated in the problem. With these solutions and score in the Problem Solving Test, Student B was considered one of *fluent* students. Solution is shown in Figure 3.

Problem 6: Quen had some stickers. He gave 1/3 of the stickers plus 2 stickers to his brother. Then he gave 1/3 of the remaining stickers plus 4 stickers to his sister. Finally, he gave ½ of what remained plus 3 stickers to his best friend. He found that he had 5 stickers left. How many stickers did Quen have at first?

Correct Answer: The number of stickers Quen have at first is 48.

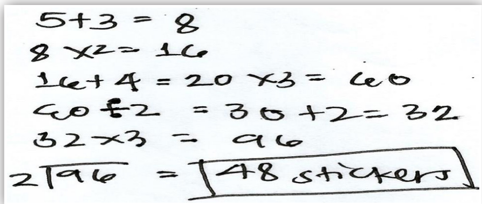


Figure 3. Solution of Student B in Problem 6.

Evidently, Student B uses the four fundamental operations in order to solve the problem. During the interview, he clarified why he divided 60 and 96 by 2, given that this specific strategy was not well-explained in his journal. He reasoned out “Yong 96 ay divided by 2 kasi pag binigay ‘yong 1/3, yong natira ay 2/3 ng stickers.” [96 must be divided by 2 because giving 1/3 to another person would leave 2/3 of the stickers.] He also added “Dahil nag-work backward ako kaya kinuha ko ang reciprocal ng 2/3 at ‘yon ay 3/2.” [Because I did working backward, I would have to get the reciprocal of 2/3; that is, 3/2.]

The solutions and verbal response reveal that Student B applied the problem solving strategy (working backwards) and the corresponding operations correctly. However, Student B overlooked the concept of “the greatest possible number of questions” in Problem 2. The following presents the solution of Student B.

Problem 2: A mathematics quiz consists of 50-multiple choice questions. A correct answer is awarded 5 marks and 2 marks are deducted for a wrong answer while no mark is awarded or deducted for each question left unanswered. If a boy scores 172 marks in the quiz, what is the greatest possible number of questions he answered correctly? Explain how you worked it out.

Correct Answer: The greatest possible number of questions that the boy answered correctly is 38.

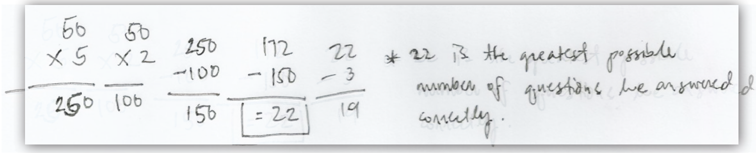
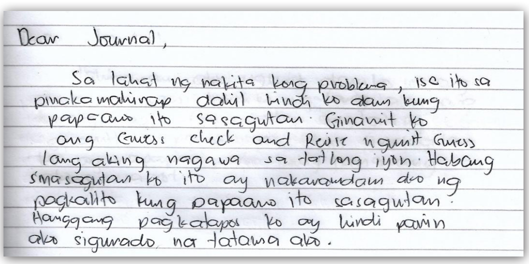


Figure 4. Solution of Student B in Problem 2.

Figure 4 shows that Student B used multiplication and subtraction in order to get the correct answer. However, he failed to verify his answer because he was confident with his solutions after doing several computations. Moreover, it seems that he did not analyze the problem completely because his solution was entirely different from the right one. His answer is 22, while the correct answer is 38.

Only one student (Student C) is regarded the *least fluent.* It can be observed in his solution that in some instances he got the answer correctly, but was unable to explain the details of the solution either in his journal or in the interview. He tried to answer the six problems, but mostly used incorrect procedures. This may be due to the reason that Student C does not have adequate knowledge of the concepts related to the task. In fact, in many of his solutions, Student C was not able to illustrate what are expected of him to successfully solve the given problem as reflected in his journal (Figure 5).


Dear Journal,

Sa lahat ng nakita kong problema, ise ito sa pinakamahirap dahil hindi ko alam kung papaano ito sasagutan. Ginamit ko ang Guess check and Revise ngunit Guess lang aking nagawa sa tatlong iyon. Habang sinasagutan ko ito ay nakaramdam ako ng pagkalito kung papaano ito sasagutan. Hanggang pagkatapos ko ay hindi parin ako sigurado na tatama ako.

Figure 5. Journal of Student C in Problem 4.

This data imply that, Student C experiences difficulty in solving problem 2. He wrote “Hindi ko alam kung paano ito sasagutan.” [I do not know how to answer this problem.] To clarify his answer further, it was asked during the interview how he was able to write down the three labels. He just kept on saying that it was a “nosebleed” problem to him. He even said “Nag-guess ako at inisip ko lang ‘yong labels” [I used guessing and thought of those labels.] Further, he said “Hindi ko ma-explain kasi naguguluhan ako.” [I could not explain it well because I am totally confused.]

On Flexibility

Eight students are regarded as most flexible, and who were able to apply two or more strategies leading to the correct solution/answer. They were able to shift from one way of thinking to another and extract solutions in different categories.

An example of a most flexible student is Student D. She was able to show at least two strategies effectively in solving Problem 1 (Figure 6).

Problem 1: A dartboard has sections labelled 2, 5, 9, 13 and 17. Justine scored exactly 356. What is the minimum number of darts he might have thrown? How did you get your answer?

Correct Answer: The minimum number of darts he might have thrown is 23.

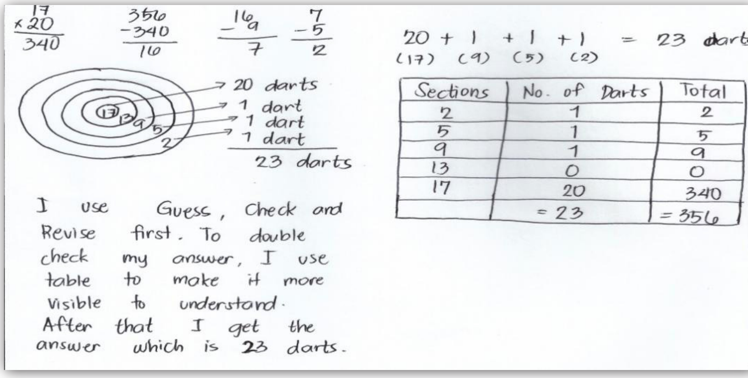


Figure 6. Solution of Student D in Problem 1.

Notably, instead of just using some operations, Student D drew a figure to concretize the situation, and made a table of values to verify if her answer is correct. She also thought of guessing numbers and simplifying it to double check her answer. It was evident in her solution,(Figure 6), when she said that *"I use Guess, Check and Revise first. To double check my answer, I use table to make it more visible to understand. After that I get the answer which is 23 darts."* She even stated in the interview that *"Guessing kasi 'yong pinakamadaling strategy tapos i-check lang kung tatama ba doon sa given."* [Guessing is the easiest strategy to apply, and then checking if it will satisfy the given.] Evidently, Student D executed at least two strategies correctly including Drawing a Figure, Guess, Check and Revise, Using Table, Chart or List, and Compute or Simplify.

Respondents who had a total score of 11-20 are considered *flexible.* They applied only one strategy leading to the correct solution/answer. Also, there were instances that these students were able to use one strategy successfully, but further explanation and clarity in their works are not visible both in their solutions in the journal and explanations during the interview.

An example of this scenario is the work of Student E (Figure 7).

Problem 2: A mathematics quiz consists of 50-multiple choice questions. A correct answer is awarded 5 marks and 2 marks are deducted for a wrong answer while no marks awarded or deducted for each question left unanswered. If a boy scores 172 marks in the quiz, what is the greatest possible number of questions he answered correctly? Explain how you worked it out.

Correct Answer: The greatest possible number of questions that the boy answered correctly is 38.

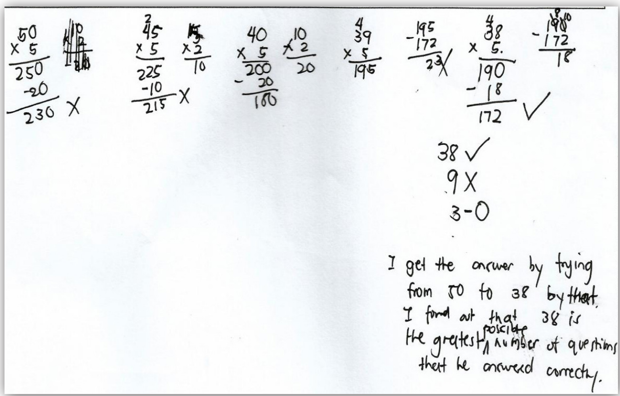
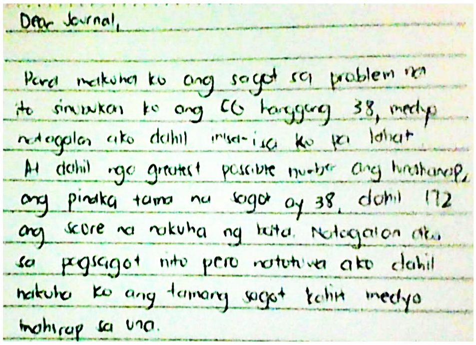


Figure 7. Solution of Student E in Problem 2.

In solving the problem, Student E used guess and check. Although she had a very long solution to initially arrive at a wrong answer of 23, she did not lose patience. This claim was supported by what she wrote in her journal as seen in Figure 8. She emphasized *"medyo natagalan ako dahil inisa-isa ko pa… pero natutuwa ako dahil nakuha ko ang tamang sagot kahit medyo mahirap sa una"*. [It took me a hard time… but I was happy because I got the correct answer even I find it difficult at first].



Dear Journal,

Para makuha ko ang sagot sa problem na ito sinubukan ko ang 66 hanggang 38, medyo natagalan ako dahil isa-isa ko pa lahat. At dahil nga greatest possible number ang hinahanap, ang pinaka tama na sagot ay 38, dahil 172 ang score na nakuha ng kita. Natagalan ako sa pagsagot nito pero natutuwa ako dahil nakuha ko ang tamang sagot kahit medyo mahirap sa una.

Figure 8. Journal of Student E in Problem 2.

Furthermore, only one student was categorized as *least flexible*. In most of the problems, this student was not able to arrive at the correct answer in applying only one strategy such as Guess, Check and Revise, Compute or Simplify. Nothing in her solutions would show that she validated her answer nor she used other ways of getting the correct answer. This increased her chances of arriving at an incorrect answer. These facts were triangulated during the interview. She stated in her interview *"nahihirapan akong intindihin yung problem kaya hindi ko din magawang madouble check kung tama o mali ang sagot ko"*. [I could not understand the problem completely so I did not even bother to check if my answer is right or wrong.] She further added *"ang hirap kasing intindihin talaga"*. [It's really very difficult to understand.]

In sum, though most respondents understand the problem, they seem to lack the knowledge of different problem solving heuristics that could help them identify the solution leading to the correct answer. This result affirms the claim of Yimer and Ellerton (2009) that majority of students are capable of understanding the problems but they lack the skill to create a procedure that will guide them to the correct solution. Therefore, in classroom drills, students should be given the opportunity to use their own strategy, instead of imposing only one strategy to solve. It is suggested that the general methods and strategies in solving mathematical problems be taught to students, but the strategies to apply must be left for the students to explore. To make problem solving a habit of the students, exercises should be provided at frequent intervals. Accordingly, teachers should encourage the students to be flexible and critical in problem solving. Villareal (2014) explains that for teachers to teach problem-solving heuristics effectively, teachers need to be open-minded.

On originality

Students had difficulty in generating unique and original solutions, thus making originality the least component. Only three students fall under most original, 13 under original and the remaining 14 students under least original.

The three students who showed most original solutions created unique and effective solutions that helped them get and verify the correct answers. These students fully understood the task by coming up with insightful thoughts and in some cases, logical extensions to the problem. An example of these works are as follows.

Problem 3: A piece of paper is 60cm by 40cm. It is to be divided into the biggest possible squares without any material wasted. How many squares can be formed?

Correct Answer: The number of squares that can be formed using the dimensions given is 6.

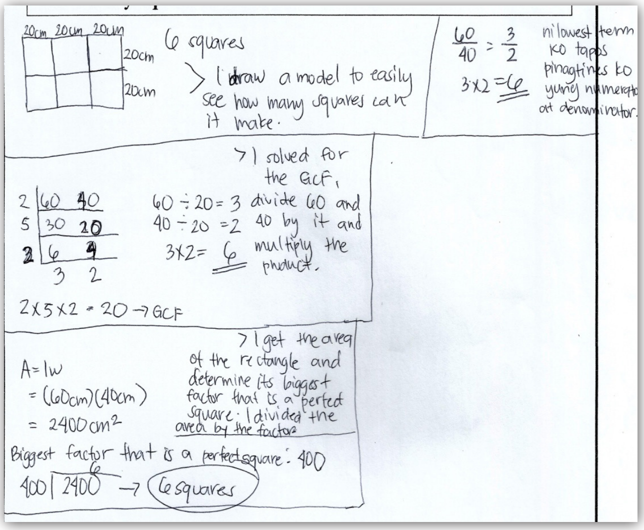


Figure 9. Solution of Student E in Problem 3.

The most common solution of the respondents in solving Problem 3 is computing the greatest common factor of the given numbers – 60 and 40. In fact, Student E exhibited solutions that were not illustrated by the rest of the respondents. These are drawing the figure to concretize the situation and using the concept of area and dividing that area by its biggest factor (perfect square). Evidently, Student 2 understood the problem very well.

Thirteen students, who earned a total score of 11-20 in terms of originality, (*original* category) may have shown two or more strategies but some of the strategies employed were also used by the majority of respondents. For instance, Student F used Guess and Check when it was reiterated *"kasi naisip ko na 7 yung colors kaya nag-add ako ng 7. Tapos tsek ko kung tama"*. [*I added 7 because there are 7 different colors. And I checked if my answer was correct*]. Likewise, another strategy was used as evidenced in the chart that was found in the solution. When asked to explain the solution below the chart, an immediate answer was "*Inisip ko lang po na para at least* *three beads ginawa ko pong 5 trials."* [*In order to get at least three beads, I made it 5 trials*]. Notably, guess and check and using tables and charts were also used by other students.

Majority of the students exhibited least performance in originality because they stopped in generating other strategies after arriving at the correct answer. Some of them were not able to earn points since they failed to get the correct answer in some problems. Almost all students used similar strategies in solving the non-routine problems. Even though these students were successful in getting the correct answer, they still got a lower score in originality because they were not able to show a different solution as compared to other respondents. Though Leikin (2009) mentioned that originality appears to be the strongest component of creativity, most of the respondents in this study failed to give a unique solution, hence the low performance in originality. But notably, three students performed best in the three components of mathematical creativity, and this confirms Lev and Leikin's (2013) idea that originality is of "gift type". Training and exposure to different competitions are probably the contributing factors in their excellent performance in these three levels of mathematical creativity. Kumar (n.d.) suggested that as creative abilities of the students may be enhanced through training, it should be the legitimate function of the education system to provide such training to foster creativity. Moreover, in many instances, students show solutions that seemed incomplete and vague. They were able to clarify and defend their answers through journals and interviews, and were also given the chance to reflect on the correctness of their answers and solutions when asked to explain what they wrote in their journals. After the interviews, students appear to understand the problems and how to solve them. This finding supports the recommendation made by Laset (2004) that educators can use journals and interviews as assessment tools because reasoning, thoughts and feelings are not apparent in the students' solutions.

As revealed in the journals and verbatim response, students believe that they could hardly come up with original solutions if they did not understand the problem well. Moreover, it was really surprising when two respondents mentioned that they could use two of their five senses namely sense of sight and smell to determine the answer. This may be attributed to the students' failure to understand the problem, recall the necessary concepts in probability and logic and find all the slices of information that need to be intertwined to carry out the entire task. Accordingly, it is not enough to learn the mathematical content, but to use this content to develop the thinking skills and solve problems (Dendane, 2009). This was further affirmed by Siniguian (n.d.) who reiterated that when a student is unable to recall basic math facts, he/she may have difficulty in maintaining correctness in the entire mathematical task leading to incorrect solutions and answers.

Conclusions and Recommendations

The main objective of the study is to describe the mathematical creativity of students in solving non-routine problems. It was found out that these students are moderately creative in all three components – fluency, flexibility and originality, but they performed best in fluency and least in originality. This finding confirms the trend derived from previous studies on the description of mathematical creativity of participants who are in high achieving schools, either in private or special science high schools. It also fills the gap in those previous studies as the result of this study seems to imply that non-routine problems are not only for high ability students but for average students as well. Therefore, students attending regular schools can solve non-routine problems and provide original solutions as well.

Furthermore, it was observed that students who were considered *most fluent* and *most flexible* followed step-by-step processes and showed very detailed solutions in getting the correct answer. Notably, these students were the ones involved in various mathematics trainings and contests. It may be inferred that students who are exposed to different training sessions and contests have the bigger potential in attaining the highest level of mathematical creativity. Consequently, students' flexibility and fluency in solving mathematics problems may be improved by encouraging them to participate in at least mathematics trainings, if not in mathematics contests, inside or outside the school.

Similarly, it was noted that most respondents can understand the idea of the problem. However, they lack the knowledge of different problem solving heuristics that will help them identify the solutions needed to obtain the correct answer. Thus, to enhance problem solving in mathematics, it is suggested that students be encouraged to use any strategy that they feel appropriate to the problem and suited to their ability, instead of imposing only one strategy to solve. While it is important to teach varied methods and strategies in solving problems in mathematics, it is equally important that students are given opportunities to employ and explore the specific strategies and solutions that they prefer. Presenting original solutions may be encouraged and students doing so may be recognized as this may motivate other students to think of original solutions as well.

Remarkably, it was noticed that most of the students had difficulty in answering problems that require the knowledge of basic probability concepts. Furthermore, it was evident in the solutions that most errors could have been avoided with the proper application of fundamental operations on basic mathematical concepts such as fractions. Teachers should therefore be more creative in teaching the basics of mathematics such as fractions and probability concepts given that students' failure in having mastered these concepts will affect their problem-solving skills in the future. Moreover, it was evident in the study that students showed solutions that seemed incomplete and vague. However, their ideas were revealed and clarified in the journal entries and group interviews. These instruments used together can provide thorough and balanced assessments.

Nevertheless, this study is limited to just 30 students of a regular high school, it is hereby recommended to consider further researches on mathematical creativity of students with a larger number of participants in more regular schools. A comparison of mathematical creativity and problem solving heuristics of students in regular and high-performing schools can also be done. Researchers may also include speed as a variable of mathematical creativity.

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